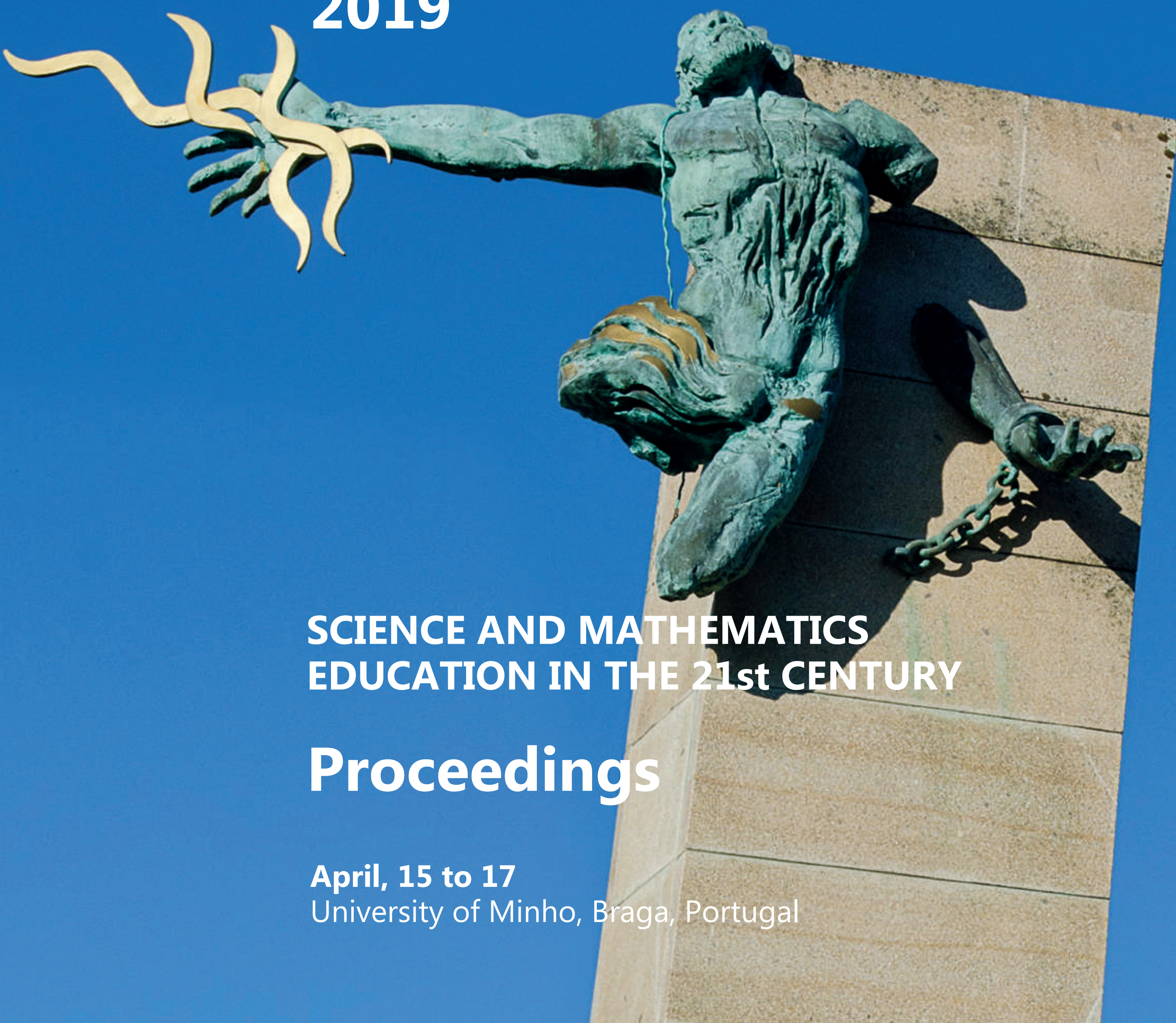
ATEE

Association for Teacher Education in Europe

WINTER CONFERENCE 2019



SCIENCE AND MATHEMATICS EDUCATION IN THE 21st CENTURY

Proceedings

April, 15 to 17

University of Minho, Braga, Portugal

ATEE WINTER CONFERENCE 2019

Science and mathematics education in the 21st century

PROCEEDINGS

ATEE Winter Conference 2019 – Science and mathematics education in the 21st century

Proceedings

Published 2019

ISBN: 9789082506549

Editors: Laurinda Leite, Elizabeth Oldham, Luísa Carvalho, Ana S. Afonso, Floriano Viseu, Luís Dourado, Maria Helena Martinho

Cover design by Nicolau Moreira and Paulo Oliveira

Published by:

ATEE (Association for Teacher Education)
Brussels, Belgium

And

CIEd (Research Centre on Education - UID/CED/01661/2019**)**
Institute of Education, University of Minho
Braga, Portugal

Introduction

The University of Minho, Braga, Portugal, hosted the 2019 Winter Conference of the Association for Teacher Education in Europe (ATEE), which took place from 15th to 17th April.

The ATEE Winter conference is one of the three ATEE conferences which are organised every year, the others being the Spring conference and the Annual conference. The Winter conference is usually organized by one (or more) of the several ATEE Research and Development Communities (RDCs). This Conference was organized by the RDC on Science and Mathematics Education, which is one of the longest-standing and most active RDCs.

The Conference focused on Science and Mathematics Education in the 21st Century. It is fully accepted that, nowadays, Science and Mathematics Education is an area of interest to policy-makers worldwide. The area is relevant not only for the ongoing scientific and technological development of modern, globalized and digital societies, but also for citizenship education and the sustainability of the Planet. Moreover, it contributes to the full development of the individual learner. Science and Mathematics teachers, teacher educators and researchers can therefore help to make a difference beyond the classroom.

However, Science and Mathematics Education is under pressure. It has been unable to develop good levels of literacy and numeracy, to lead enough youngsters to engage in science and technology careers, and to overcome many people's dislike and even fear of the subjects. Thus, the Conference aimed at promoting forward-looking approaches that combine engagement and enjoyment with effectiveness in developing knowledge and skills, and hence fostering ways of overcoming the challenges that the area has been facing. The Conference was a forum to enhance deep and multicultural discussions on issues like: innovative approaches to teaching science and mathematics; technologically enhanced science and mathematics education; science and mathematics education and the STEM agenda; science and mathematics education in multicultural and inclusive schools; science and mathematics teacher education in a changing world; and 21st century assessment in science and mathematics education.

Two Keynotes addressed and elaborated on the conference theme. A Panel offered an opportunity for getting some insight into ways forward with regard to the challenges that science and mathematics education face in the present century. Two Plenary presentations highlighted European and Portuguese policies for Science and Mathematics Education.

Over seventy papers and posters were presented at the Conference. The abstracts of the paper and poster proposals were blind refereed by at least two members of the Conference Academic Committee and many of them were reviewed and improved by their author(s) before being accepted. Altogether, they offered a rich and multifaceted picture of the Conference theme.

Twenty-one full papers (fourteen related to paper presentations, and seven to poster presentations) were submitted and fifteen (ten related to paper presentations and five to poster presentations) were accepted for inclusion in the Proceedings. It should be noted that, even though full papers were submitted to a double peer review process, the content and ideas conveyed by them as well as the language used are the authors' own responsibility.

The organisers of the Proceedings would like to express their gratitude to the Conference Academic and Organising committees and to all the people, institutions and organisations that, in various ways, sponsored the Conference. However, special thanks are due to ATEE and to the RDC on Science and Mathematics Education for trusting the conference organizers, and to the University of Minho for hosting and supporting the Conference.

Table of contents

Science teachers' in-service education to enhance students' critical and creative thinking

Ana Sofia Sousa & Rui Marques Vieira

Centro de Investigação em Didática e Tecnologia na Formação de Formadores (CIDTFF), Universidade de Aveiro, Portugal

anasofiasousa@ua.pt | rvieira@ua.pt

Abstract: Science and Technology have a major impact in nowadays people's lifestyle. Therefore, science education plays a vital role in students' preparation for the challenges and possibilities that the unknown future presents. Its addition is considered a fundamental setting for thinking abilities that encourage innovative solutions and reflective reasoning, particularly critical and creative thinking (CCT), closely linked, according to international education guiding principles, with a conscious and responsible citizenship.

However, some argue that the didactical proposals used in science classrooms are, in general, distant from those guidelines providing little development of CCT in a stated and structured manner. Since teacher's pre-service education does not include instructions on how to promote CCT, it is important to provide opportunities, through specific in-service teacher's education, for them to become prepared to effectively enhance their student's CCT. This work describes a theoretically based proposal related to an in-service education programme aiming to enable and support science teachers in the promotion of students' CCT and discusses some educational implications.

Keywords: creative thinking; critical thinking; in-service education; science education

Introduction

Modern societies demand crosscutting practical and intellectual abilities that are of central importance to all the dimensions of human life. In this sense, critical and creative thinking (CCT) are more important than ever as two productive, deliberate and transversal ways of thinking that are vital to effective learning and to an active and informed citizenship.

The strong natural link of complementarity that can be inferred between "critical thinking" and "creative thinking" enables the making of informed decisions, the prediction and logical assessment of the impact of those decisions, the autonomous development of adaptive and innovative ideas to solve problems and the understanding of socially contextualized scientific processes and phenomena (Paul & Elder, 2014). Consequently, CCT are important tools not only for meaningful learning but also for preparing students to live and succeed in today's globalized economies.

Despite the widespread acceptance of CCT as one of the key learning outcomes for science education, there seems to be little evidence that this thinking abilities are taught in the gradual, intentional, explicit and systematic way advocated by research (Coil, Wenderoth, Cunningham, & Dirks, 2010; Vieira, 2018). Among different reasons, this is also perhaps due to the fact that fostering CCT in science classes requires a specific education which, in general, teacher education, whether the initial or the ongoing continuous training, offered to teachers, does not embrace (Vieira & Tenreiro-Vieira, 2016). Consequently, this communication presents a theoretically based proposal related to an in-service education programme that prepares and qualifies teachers to respond to the need of promoting CCT since the first years of schooling. Accordingly, this education programme, a part of an undergoing PhD research, consists of a thirteen hours course with a presencial work regime that emphasizes reflective collaborative work and considers the findings of earlier researches. These researches point at the importance of providing professional learning opportunities throughout teachers' long-term careers so they can remain confident and motivated as agents of change in the classroom (Loucks-Horsley & Matsumoto, 1999). Its implementation aims the renewal of didactical practices in science classes to enhance students' thinking abilities, useful scientific knowledge and deepen their engagement in the learning process.

Aim

There is a broad consensus that science education is crucial for citizen's personal development and for the social, economic and political progress of countries. At the same time, the fact that Science and Technology currently guide the evolution of societies requires the development of thinking capacities such as CCT that enable responsible and democratic civic intervention. As stated, CCT are considered a priority educational main goal in a growing number of educational curricula in countries around the world and is currently, for instance, a mandatory recruiting requirement by innovative companies (OECD, 2018; World Economic Forum, 2016). Thus, the natural relationships established between Science, Technology, Society and Environment elect science education as a privileged context for the development of CCT since the earliest years of schooling.

In order to respond to the already outlined teacher's formation needs, this work presents a proposition of an in-service education programme to prepare and qualify science teachers to respond to the difficult task of promoting students' CCT that has been accredited by the Portuguese Scientific-Pedagogical Council of Continuing Education for Portuguese Basic Education's teachers. This education programme intents to confront its target groups, teachers of the 1st and 2nd Cycles of Basic Education (from first to sixth grades), with situations that can contribute to: i) make teachers aware of CCT importance for themselves and for students in the response to the contemporary and future world's challenges; ii) equip teachers with the appropriate and necessary qualifications for the systematic and structured promotion of CCT abilities in science classes based on their professional, personal and social development; iii) reconfigure their conceptions and enhances the adoption of didactical practices that incorporate CCT effective development in their science classes.

Theoretical framework

Science education and critical and creative thinking

Constant societal changes and the uncertainty and unpredictability that characterize the future poses important challenges for schools in general, and for science teachers in particular, due to the strong influence of Science and Technology on contemporary societies. Consequently, it is crucial that students are prepared to be able to evaluate, make socially conscious decisions and express informed opinions capable of questioning and intervening in the resolution of scientific, technological and environmental issues that are crucial for the future of humanity. To be able to intervene as rational, conscious and democratic citizens, students need to develop thinking abilities such as CCT.

"Critical thinking" and "creative thinking" are concepts that have been defined within different fields of knowledge (e.g., philosophy, psychology, …) by various authors whose contributions have influenced current education policies aiming to educate citizens who can reflect and draw independent conclusions about the information they receive and the environment they live in (Torres Merchán & Solbes Matarredona, 2016; Phan, 2010; Runco, 2007). In this framework, "critical thinking" can be defined, for example, as a rational and reflective process that integrates abilities (e.g., "analyse arguments", "judge the credibility of a source" or "make and judge value judgments"), dispositions (e.g., "consider seriously other points of view than their own" or "try to be well informed"), knowledge (e.g., "scientific principles and explanations" or "Science History") and criteria (e.g., "accuracy" or "precision"), focusing on deciding what to believe, finding explanations, making informed decisions and solving challenges (Ennis, 2011; Franco, Vieira, & Saiz, 2017; Vieira, 2018). In its turn, "creative thinking" can be described as an intentional, individual or collective multidimensional thinking, involving abilities such as "originality" or "fluency", to manipulate symbols or objects in order to generate, test, and apply alternative and/or original ideas for problem solving (Torrance, 1966; Sternberg, 2012).

In addition to the diversity of existing conceptions and the conceptual complexity that surrounds CCT, there are other reasons that reflect the inherent difficulties linked with its promotion in the classroom, such as the fact that "critical thinking" is neither natural nor spontaneous or the results that show that "creative thinking" has been decreasing significantly over the last three decades (Kim, 2011; Tenreiro-Vieira & Vieira, 2019). In fact, most of the human thinking is "(…) biased, distorted, partial, uninformed or down-right impaired" (Paul & Elder, 2014, p. 6) and when people need to solve problems they tend to look for "ready-made solutions" (Ülger, 2016, p. 695). To reverse this scenario and due to its educational importance, there have been efforts to integrate the promotion of CCT into many countries' curricula (e.g., Australia or United States of America). However, mobilizing students' CCT is not an easy task and requires a systematic and intentional approach and the use of explicitly oriented teaching strategies (Vieira & Tenreiro-Vieira, 2016; Vieira, 2018).

Besides the lack of teachers' education in topics related to CCT, there are other reasons, such as misconceptions due to the fact the meaning ascribed to these concepts is hardly explicit (Runco, 2007; Vieira, Tenreiro-Vieira, & Martins, 2011). Nevertheless, teachers play a decisive role in the process related to the development of students' CCT in spite of this increasingly complex task given the diversity of students, their needs and the demands of a constantly changing society. In this logic, is necessary to recognize teachers' needs and provide training opportunities that include a reflective component, considered as essential for the renewal of conceptions and practices that can improve teaching and learning processes.

Teachers' professional development

Professional development of teachers is advocated and recognized as a vital and decisive element of the teaching process and, consequently, of meaningful learning. Continuing teachers' in-service formation is meant to complement their initial education, should enhance knowledge-related factual and procedural development about the contents to be taught and improve their pedagogical practices for the benefit of students (Coldwell, 2016).

Nevertheless, it is also important to note that the effective professional development will only be possible if teachers recognize that their conceptions, knowledge and didactical activities need renewal, new learning and commitment to the students and the community in which they operate.

Science teachers in-service education programme

As stated, the present science teachers in-service education programme, that will take place this year in October, will begin by characterize teachers' CCT concepts, as well as by identifying their formation needs and the resources available in their schools. The analysis of the information collected will allow the tailoring and adaptation of the contents and working methods to the specificities of the teachers involved in order to encourage learning' effectiveness and its transferability into their pedagogical practices.

Methodology

The methodology selected to this in-service education programme has a highly practical and participative approach that comprises learning activities similar to those teachers will use in their classrooms, simulations of real-context situations and best practices' exchange. It also embraces collaborative work among teachers highly recommended as a crucial element of professional development due to a set of benefits such as, among others, improving teachers' response to problems, frustration and difficulties, promoting new ideas and encouraging reflexion on professional practices (Wenger, McDermott, & Snyder, 2000).

Accordingly, and from an organizational point of view, this science teachers' in-service education programme follows the guiding principles of the process of professional teaching development highlighted by Loucks-Horsley and Matsumoto (1999) focusing on:

i) learning (creating cognitive dissonance to promote confrontation between their existing beliefs and practices and new learning and knowledge);
ii) knowledge (valuing the experience of teachers and offering opportunity, time and support for them to (re)build their conceptions and deepen the knowledge about their pedagogical practices);
iii) collaborative work (promoting effective individual and joint learning experiences and the incorporation of new systematic and explicit practices consistent with the new knowledge and the constructive critical feedback given by the rest of the group).

Therefore, all the sessions will emphasize teachers' collaborative work, promote discussion and reflection and also encourage them to share their professional experience and learning. In this line of thought, all the work proposals will be focus on the development of the participant's CCT through "hands-on" activities performed in small groups (three to four trainees) followed by moments of individual and/or collective reflexive processes focused on the strengthen and deepening of teachers' didactical knowledge on CCT promotion.

Main objectives

In accordance with the presented principles that emerged from literature and research, some objectives were defined in order to present an educational programme capable of transform teachers' concepts and practices. To this extent, the main objectives of this science teachers' in-service education programme are:

i) to provide opportunities for confrontation between teacher's current conceptions and didactical practices and the orientations provided by the research carried out in the field of Science Education by:
   - raising awareness about CCT's importance for themselves and for students' learning and overall formation;
   - broadening conceptual clarification about CCT;
   - inspiring reflection on the role of teachers and the professional skills needed to promote CCT in science classrooms;

ii) to disseminate teaching/learning' strategies that effectively promote the development of students' CCT and a meaningful science learning by:
   - supporting the appropriation and integration of systematic didactical practices that explicitly and intentionally promote CCT;
   - planning science classes that effectively promote and assess CCT;
   - providing an open space for work and discussion that integrates the professional experience of the participants and endorses the development of the necessary skills for the implementation of CCT teaching / learning strategies in science classes;
   - helping teachers develop their own thinking capacities (e.g. critical thinking, creative thinking, ...).

Content

This science teachers' in-service education programme has a flexible structure so it can be adapted to meet the specific and relevant needs arise during the sessions and maximize its teaching-learning potential. Therefore, the topics are organized in three main axes, namely:

(1) Conceptual clarification
   - What are "critical thinking" and "creative thinking"?
   - Why are they important?
   - Factors, difficulties and obstacles associated with the promotion of CCT in the classroom

(2) Classroom' culture that enhances the development of CCT
   - Principles of implementation
   - The role of the teacher
   - The role of students

(3) Strengthening CCT in science education?
   - Resources and CCT' teaching/learning strategies
   - Planning classes that effectively promote CCT
   - Assessing students' CCT

Assessment

To ensure the quality of education and the adaptation of the in-service education programme to the participants' needs, the assessment of the trainees will be based on their performance in the whole-group face-to-face sessions and on the writing of critical reflections about the impact of the in-service education programme on their conceptions and teaching practices and also on their personal, professional and social development.

Concluding remarks

In the pursuit of preparing students for the challenges that lies ahead, focus must also be placed at the improvement of critical and creative teaching. In this sense, it is important to attend to the necessary updating of teachers within the scope of Science Education that enables confrontation and adaptation to the constant changes that occur in educational contexts and enhances a science education that promotes structuring learning.

With this in-service education programme, teachers are expected to recognize CCT importance on students' learning and overall formation. In addition, it is also expected that teachers understand the professional abilities needed to promote these thinking abilities and how to systematically integrate in their practices some explicit didactic strategies that intentionally promote and assess students' CCT in science education.

In summary, promoting CCT in the classroom requires an open classroom and a new teacher's attitude to facilitate the construction of transforming knowledge that improves learning outcomes and prepares students to real life challenges. In fact, CCT are related to the building of knowledge, the development of important abilities and the clarification of believes and values that guide actions. From this, it can be deduced CCT relevance in the classroom and its importance for a conscious citizenship. However, for this to become a reality in science classrooms is mandatory to provide teachers with the specific education and support. The proposition presented, in the form of a science teachers' in-service educational programme, aims to respond to teachers' current needs and contribute to empower them to promote their students' CCT as advocated by international education guidelines.

Implications for science education

The importance of CCT in science education is widely recognized. Its potential to the overall education of citizens able to strive in a plural and unpredictable world. Along with the need for more research, Science Education must take into account the existent scientific results provided by CCT researchers that point to the effectiveness of some didactical strategies when used in a targeted, systematically and intentional manner.

However, developing students' CCT abilities requires specific teachers' education programmes and the development of their own personal and professional abilities. In this line, there is a valuable formative importance for teachers in sharing, analysing, discussing and reflecting about their convictions and didactical practices, as well as in adopting a more critical and creative approach supported by the usage of technology in the classroom. In this sense, the science teacher's education course presented represents a contribution for this formation gap and for the effective development of this educational goal in science classrooms. As a result, teachers can contribute to improve students' construction of meaningful scientific knowledge and their learning outcomes as well as to be better prepared for a responsible, active and democratic citizenship.

Acknowledgments

This work was supported by National Funds through FCT - Fundação para a Ciência e a Tecnologia, I.P. under grant [SFRH/BD/130582/2017] and under project [UID/CED/00194/2019].

References

Coil, D., Wenderoth, M., Cunningham, M., & Dirks, C. (2010). Teaching the process of science: Faculty perceptions and the effective methodology. *CBE Life Sciences Education, 9*(4), 524-535.

Coldwell, M. (2016). Exploring the influence of professional development on teacher careers: A path model approach. *Teaching and Teacher Education, 61*, 189-198.

Ennis, R. (2011, May). *The nature of critical thinking: An outline of critical thinking dispositions and abilities.* Paper presented at Sixth International Conference on Thinking at MIT. Cambridge, MA. Retrieved from https://education.illinois.edu/docs/default-source/faculty-documents/robert-ennis/thenatureofcriticalthinking_51711_000.pdf

Franco, A., Vieira, R., & Saiz, C. (2017). O pensamento crítico: As mudanças necessárias no contexto universitário. *Revista de Estudios e Investigación en Psicología y Educación, Vol. Extr. (7),* A7-012 - A7-016.

Kim, K. (2011). The creativity crisis: The decrease in creative thinking scores on the Torrance tests of creative thinking. *Creativity Research Journal*, *23*(4), 285-295.

Loucks-Horsley, S., & Matsumoto, C. (1999). Research on professional development for teachers of mathematics and science: The state of the scene. *School Science and Mathematics*, *99*(5), 258-271.

Torres Merchán, N., & Solbes Matarredona, J. (2016). Contribuciones de una intervención didáctica usando cuestiones sociocientíficas para desarrollar el pensamiento crítico. *Enseñanza de las Ciencias*, *34*(2), 43-65.

OECD - Organisation for Economic Co-operation and Development (2018). *Teaching, assessing and learning creative and critical thinking skills in education*. Retrieved from http://www.oecd.org/education/ceri/assessingprogressionincreativeandcriticalthinkingskillsineducation.htm

Phan, H. (2010). Critical thinking as a self-regulatory process component in teaching and learning. *Psicothema*, *22*(22), 284-292.

Paul, R., & Elder, L. (2014). *Critical thinking: Tools for taking charge of your professional and personal life.* USA, New Jersey: Pearson Education.

Runco, M. (2007). *Creativity - Theories and themes: Research, development and practices.* Londres: Academic Press.

Sternberg, R. (2012). The assessment of creativity: An investment-based approach. *Creativity Research Journal, 24*, 3-12.

Tenreiro-Vieira, C., & Vieira, R. (2019). Promover o pensamento crítico em ciências na escolaridade básica: Propostas e desafios [Promoting critical thinking in science of basic school: Proposals and challenges]. *Revista Latinoamericana de Estudios Educativos, 15*(1), 36-49.

Torrance, E. (1966). *The Torrance tests of creative thinking-norms-technical manual research edition-verbal tests, forms A and B - figural tests, forms A and B.* Princeton: Personnel Press.

Ülger, K. (2016). The relationship between creative thinking and critical thinking skills of students. *H.U. Journal of Education*, *31*(4), 695-710.

Vieira, R. (2018). *Didática das ciências para o ensino básico [Didactics of Science in Basic Education]*. Faro: Sílabas & Desafios.

Vieira, R., & Tenreiro-Vieira, C. (2016). Teaching strategies and critical thinking abilities in science teacher education. In G. Gibson, (Ed.) *Critical Thinking: Theories, methods and challenges* (pp. 77-98). New York: Nova Science Publishers.

Vieira, R., Tenreiro-Vieira, C., & Martins, I. (2011). Critical thinking: Conceptual clarification and its importance in science education. *Science Education International, 22*(1), 43-54.

Wenger, E., McDermott, R., & Snyder, W. (2002). *Cultivating communities of practice*. Boston, Massachusetts. Retrieved from http://cpcoaching.it/wp-content/uploads/2012/05/WengerCPC.pdf

World Economic Forum (2016). *The future of jobs: Employment, skills and workforce strategy for the fourth industrial revolution.* Retrieved from http://www3.weforum.org/docs/WEF_Future_of_Jobs.pdf

Classroom experience as a factor in encouraging undergraduate mathematics students to consider a career in teaching: A case study

Elizabeth Oldham, Donal O'Donovan & Megan Colhoun
Trinity College Dublin, the University of Dublin, Ireland
eoldham@tcd.ie | don@maths.tcd.ie | colhoum@tcd.ie

Abstract: Shortage of mathematics teachers is a comparatively new phenomenon in Ireland, causing pressure to encourage undergraduate students of mathematics to take up careers in teaching. This paper reports on a response in one Irish university: providing an optional module on Mathematics Education, during which students spend time helping in school classrooms or similar settings ("classroom experience"). Mathematics students in the university were surveyed to find their perceptions of teaching and (for module participants) their reactions to the module and in particular to the classroom experience. Most respondents from the module - especially those who were able to interact substantially with school students - reported that they enjoyed the classroom experience and were more likely than before to consider teaching at some stage of their careers. The approach might be transferrable to other subjects and settings.

Keywords: classroom experience; mathematics teacher supply; perceptions of teaching

Introduction

Mathematics education in the 21st century can profit from research findings over preceding decades, and from increased availability of resources especially in the area of technology; but above all it will depend on *teachers*: teachers who will implement the research, employ the technology appropriately, and develop a love of mathematics in their students. Teacher supply is therefore a key issue in mathematics education. Shortage of mathematics teachers has been a widespread problem internationally; however, it is a comparatively new phenomenon in Ireland, so there has been pressure recently to find new ways of encouraging appropriate candidates to take up careers in teaching.

One response, in the authors' university, has been to offer undergraduate mathematics students an optional module on Mathematics Education, during which they spend time helping in school classrooms or similar settings ("classroom experience"). A study was carried out in 2018 to address the following research questions: (1) What is the effect of the module on students' intentions to teach?; (2) What are the students' perceptions of teaching?; (3) What aspects of their classroom experience affect their perceptions of and intentions with regard to teaching?.

The present paper is the second of two based on the data collected via surveys of participating students. The first paper provided an overview of the theoretical frameworks supporting design of the module, and gave details of the resulting structure and content; it also addressed aspects of research questions 1 and 2 (Oldham, Colhoun, & O'Donovan, 2019). The second paper focuses chiefly on research question 3.

The context is set by identifying factors currently affecting teacher recruitment in Ireland and describing aspects of the Mathematics Education module germane to this paper. The theoretical framework used in the study is then outlined, drawing on literature about perceptions of teachers and teaching and also motivations for taking up a teaching career. The methodology is described, and findings are presented and discussed; conclusions are drawn, and applications to teacher education identified.

Context

Over the past decade, several factors have come together to affect teacher recruitment in Ireland. These include stricter requirements for accreditation set by the state Teaching Council, and also the increased length and consequent financial and emotional demands of teacher education programmes (Ní Dhuinn, Prendergast, & Loxley, 2019; O'Doherty & Harford, 2018). A former Professor of Education - writing in a major Irish newspaper, thus reflecting the fact that teacher shortage has become a matter for serious public discussion - emphasised that "[we] need urgent action to ensure that we continue to attract our brightest and best young people to embrace a teaching career" (Hyland, 2019).

Among the possible candidates for careers in mathematics teaching, undergraduates following mathematics courses are a natural focus of attention. The module "Mathematics Education", provided for such students in the authors' university, was introduced in 2010. It is important to note that the module is not part of a professional teacher education programme, and that participants need make no commitment to enter such a programme; hence they can be described, at most, as *prospective* teachers. In fact the chief aim in setting up the module was to make an immediate contribution to Irish mathematics education by sending students with good content knowledge to help teachers in school classrooms. However, encouraging the students to consider a teaching career was a secondary aim. The module was codenamed MA3496, the first four characters indicating its availability to mathematics students in their third and fourth undergraduate years. It has two main components: a lecture course (taught chiefly by the first author, and largely confined to the first semester of the academic year) and the classroom experience (the area of concern of the second author, and consisting of time spent in school or other classrooms during the second semester). As indicated above, the design of the module and general findings from the study are described in an earlier paper (Oldham et al., 2019). In the present paper, the chief focus is on the classroom experience component.

For this component, the original intention was that typically the undergraduates would help individual school students during "seat work" sessions, but that teachers might specify other kinds of assistance if they wished. With a larger undergraduate uptake than had been anticipated (up to 40 students in some years, whereas numbers between 5 and 15 had been expected), it was challenging to find appropriate schools and negotiate contact times that did not clash with the students' other commitments. A wider range of placements had to be used. Some students assisted with homework rather than in lessons; some were placed in college "helprooms" (tutoring undergraduate students in earlier stages of their college careers); some contributed to "access programmes" (for students coming to college from non-traditional backgrounds or by non-standard routes, and hence being given extra support); and some helped young patients in a hospital. Moreover, in recent years, students have been encouraged to organise their own classroom experience. For example, some have returned to the schools where they themselves studied, and some have made arrangements with schools near their homes to minimise travel.

It should be emphasised that the university staff do not monitor the students while on placement; monitoring would detract from the main aim of allowing the schools or teachers to organise the students' input in whatever way best suits them (the schools and teachers). Insights into the experience are therefore dependent on student self-report and on feedback from schools and teachers involved.

Theoretical framework: Motivations for and perceptions of teaching

Notably with a view to understanding the reasons for teacher shortage, ***motivations*** for taking up a career in teaching have been examined extensively in recent years. Studies that focused on potential or preservice teachers are most relevant to this paper. In such studies, motivations are often classified as "altruistic", "intrinsic" or "extrinsic" (see for example: Bergmark, Lundström, Manderstedt, & Palo, 2018; Han & Yin, 2016; Nesje, Brandmo, & Berger, 2018; Struyven, Jacobs, & Dochy, 2013; Thomson, Turner, & Nietfield, 2012). According to Struyven et al. (2013, p. 1009):

"1. *Altruistic reasons* refer to individual perceptions of teaching as a socially valuable or important job, to the desire to help children and young people succeed and to improve society.
2. *Intrinsic reason*s are reasons inherent to the job itself. Students cite intrinsic reasons when they refer to their passion and vocation for the activity of teaching children in general (e.g. 'I always wanted to teach') and their interest in using their subject matter knowledge and expertise in particular.
3. *Extrinsic reasons* are related to job characteristics not inherent to the job itself, such as long holidays, salary, status and working conditions."

While these descriptions are helpful in broadly delineating the area, the categories are not defined identically across different studies, leading to some difficulties in comparing results (Nesje et al., 2018; Watt et al., 2012). Additionally, in some studies consideration is given to other factors relevant for career choice: for example 'beliefs about teaching and perceptions of teachers and teaching' (Bergmark et al., 2018; Thomson et al., 2012).

To examine these constructs, both quantitative and qualitative approaches have been used, with the former leading to the design and use of instruments to measure various factors. One notable development has been that of the so-called "FIT-choice" model (Factors Influencing Teaching Choice model), due to Watt and Richardson (2007) and shown in Figure 1.

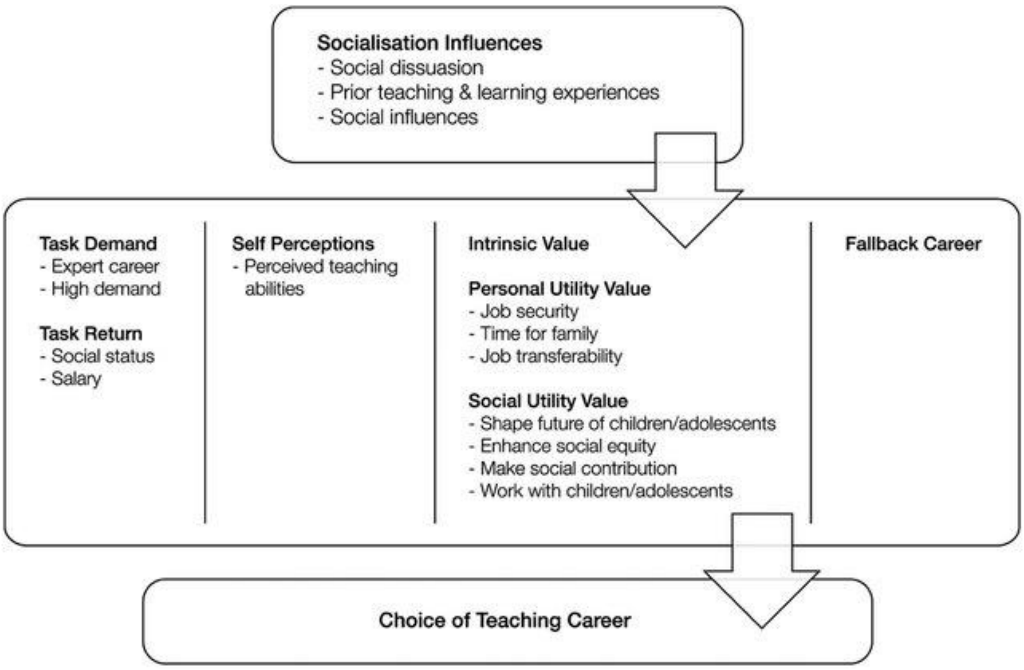


Figure 1. The FIT-choice model (Watt & Richardson, 2007)

According to this model, *Socialisation Influences* are identified as affecting *Motivation* (the large box, not labelled in the figure), four aspects of which are *Task Perceptions, Self Perceptions, Perceived Values* and *Teaching as a Fallback Career*; these aspects then influence *Choice of Teaching Career*. While the motivational aspects in the model are not identical to the categories in other literature cited above, similarities can be identified. Task Perceptions can be aligned to beliefs about or perceptions of teachers and teaching; for Perceived Values, Personal Utility relates to extrinsic motivation, while Social Utility may relate to altruism. An instrument based on the model, the "FIT-choice scale," has been widely used with preservice teachers and appears to be robust across cohorts from different countries (Nesje et al., 2018; Watt et al., 2012). Among other tools dealing with motivations for and/or beliefs about teaching, one of particular relevance is the "Reasons for Teaching" instrument, utilised by Thomson et al. (2012) in their work with students in preservice teacher education programmes.

Studies based on data collected from such instruments, and/or from interviews, have sought to identify typologies of preservice teachers, for example based on perceptions of or commitment to a teaching career (Moses, Berry, Saab, & Admiraal, 2017; Thomson et al., 2012). Such work could be useful when selecting students to enter preservice courses - especially in the happy situation when there is a surplus of eligible candidates.

Methodology

The main purpose of the study reported here was to find out if (and how) taking the module MA3496 affected participants' intention to take up a career in teaching at some stage of their lives. An additional aim was to examine views of third-year and fourth-year undergraduate mathematicians in the College (whether taking the module or not) on teaching and teachers in general. Given the focus on a particular context and set of participants - the features of participants' classroom experience being very specific to the module - the approach is that of case study research.

Research questions

For this paper, the broad research questions for the study (listed in the Introduction) can be refined as follows:

(A) What are the students' views on teaching in general and on their classroom experience in particular?

(B) What trends and/or patterns of response exist with regard to:
- Participants' classroom experience
- Their likelihood of teaching at some stage
- Their perceptions of teachers and teaching?

Instrument design and data collection

Data were collected only via questionnaires. Students are very busy during the year and are hard to contact after it has ended, so the deeper insights that might have come from interviewing even selected participants had to be sacrificed to overall convenience. Two instruments - questionnaires 1 and 2 - were designed, to be completed online (a method easy for and familiar to the students). They were based on the factors affecting teacher recruitment in Ireland, the theoretical framework on motivations for and perceptions of teaching, and the structure of and practicalities involved in the module organisation.

Questionnaire 1 formed part of the third author's undergraduate research project, and was aimed chiefly at finding if undergraduate mathematicians were interested in teaching as a career (Colhoun, 2018). The final version, after piloting, sought information from respondents on (*inter alia*):

- Their intentions as regards considering a career in teaching ("yes," "no," "maybe")
- Whether or not they were taking or had taken the module MA3496, and if so whether it had affected their intentions to teach ("less likely," "no change," "more likely"), with reasons (open response).

The questionnaire was set up using the limited version of SurveyMonkey available free to students. Links were posted for access by members of two Facebook groups spanning the target population, third-year and fourth-year mathematics students in Trinity College in the academic year 2017-18. The exact number involved was unclear, as some original members may have dropped out since the groups were established, but was around 60 (Colhoun, 2018). Data were collected during the second semester: late enough in the year for students taking the module to report at least some of its effects.

Questionnaire 2 was designed by the first two authors to investigate the research questions for the study, and also to obtain feedback that might be useful in improving the module. Items relevant to this paper sought information on

- Students' level of agreement with the statement "I might teach in a school at some stage of my career" ("strongly disagree" to "strongly agree")
- The effect of each of the following on students' intentions to teach at some stage ("more likely," "unchanged," "less likely"):
  - The classroom experience overall
  - Various aspects of the classroom experience (see Table 5 below)
- Organisational aspects:
  - Who arranged the placement (the second author or the student)
  - The setting for the experience (school classrooms, helprooms and so forth)
- Students' level of agreement with statements about their classroom experience ("strongly disagree" to "strongly agree") (see Table 3 below)
- Students' level of agreement with statements on demands and benefits involved in teaching ("strongly disagree" to "strongly agree") (see Table 4 below).

For the final section, seven of the ten items were from the FIT-choice scale and three from the Reasons for Teaching instrument, described above. Using the FIT-choice classifications, four items were intended to address Task Perceptions (Task Demand or Task Return), and five to address Perceived Values (Personal Utility or Social Utility). One of the items from the Reasons for Teaching instrument - "Teaching is a noble profession" - made no reference to utility, and so was classified with its original label of Altruism. Respondents were also given the opportunity to provide comments on the classroom experience and the module overall. The questionnaire was set up using Google forms. After the assessment results for the year 2017-18 had been published - thus obviating any worries about effects on students' marks despite the anonymity of the responses - a link was sent in an email to all people who took the module in either of the sessions 2016-17 and 2017-18 (78 people, some of whom had left college at the end of the year 2016-17).

Data analysis

The quantitative data were coded and entered into SPSS, and frequency counts generated. Exploratory analysis was carried out with regard to variables hypothesised as having explanatory power for the study: in particular, the organisational aspects (who arranged the placement and which type of setting was experienced). Comparisons were made between groups of interest (hence, for example, 'the group of students who had arranged their own placements versus those for whom it had been arranged by the second author, and the group who had had experience in school classrooms and had been able to interact with students versus groups with other kinds of experience'). Shapiro-Wilk tests of normality were run for relevant Likert-type variables, and Mann-Whitney tests used because all distributions were non-normal; for relationships among other variables of relevance, chi-square tests were carried out.

The qualitative data - the open responses - were categorised by whether, as a result of the classroom experience (questionnaire 2) or the module overall (questionnaire 1), the respondents reported being more likely to teach at some stage, unchanged with regard to their likelihood, or less likely. They were cross-tabulated with whether the respondents were positive towards going into teaching (agreeing that they would consider a teaching career or might teach in school at some stage), unsure about it, or negative about it. Patterns with regard to the classroom experience were sought, and the responses were also examined to see if and how they reflected factors identified in the theoretical framework.

Findings and discussion

For questionnaire 1, 30 responses were received from the group of around 60, giving a response rate of about 50%. Twenty of the respondents reported that they had taken or were taking the module. For questionnaire 2, there were 31 responses from the target group of 78. One person who had taken the module in 2015-16 also responded; in recognition of that person's effort, the data were included, giving a response rate of 40.5%. The groups may overlap; some people may have responded to both questionnaires.

Details of the organisation of the classroom experience were obtained from questionnaire 2. Of the 32 respondents, 13 reported that the experience had been arranged by the second author, and 18 that they had made their own arrangements; one endorsed both categories. With regard to the different settings, 24 stated that they had been in regular school classrooms and had at least some interaction with school students; the numbers reporting other types of experience were small.

Results are presented to address research questions (A) and (B) in turn. For question (A), the data are "snapshots" of views at the time of completing the questionnaires. Research question (B) refers to trends - reported changes in views as a result of taking the module - and patterns amongst the responses.

Research question (A): Students' reported views on teaching and classroom experience

With regard to the possibility of teaching at some stage, the items in the two questionnaires were worded differently, so the responses are shown separately in Tables 1 and 2. While few of the respondents endorsed the most positive category presented in each case, around one-third indicated that they might be open to some eventual involvement in school teaching (choosing "Maybe" or "Not sure"). For questionnaire 2, over half of the respondents agreed that they might teach at some stage, and only three students ruled out such involvement.

Table 1. Responses to the question "Would you consider a career in teaching mathematics?" (questionnaire 1)

Respondents	**Yes No. (%)**	**No No. (%)**	**Maybe No. (%)**	**Total number**
All	5 (16.7)	14 (46.7)	11 (36.7)	30
Taken / taking MA3496	3 (15.0)	10 (50.0)	7 (35.0)	20

Table 2. Responses to the statement "I might teach in a school at some stage of my career" (questionnaire 2)

Strongly disagree No. (%)	**Disagree No. (%)**	**Not sure No. (%)**	**Agree No. (%)**	**Strongly agree No. (%)**	**Total number**
0 (0.0)	3 (9.4)	10 (31.3)	17 (53.1)	2 (6.3)	32

The respondents' perceptions of different aspects of their classroom experience are shown in Table 3. Where the numbers responding are recorded as less than 32, the remaining students had chosen a "not applicable" response (and percentages are of those responding). Most reported positively, especially with regard to their helping the students with whom they worked. The level of strong agreement with the statements about enjoying the experience and being welcome in the school is noteworthy. However, almost a third of respondents reported being, at best, not sure if their role was clearly understood; perhaps there is a need for better communication with the schools or organisations involved.

Table 3. Level of agreement with statements about classroom experience (questionnaire 2)

Item	**Strongly disagree No. (%)**	**Disagree No. (%)**	**Not sure No. (%)**	**Agree No. (%)**	**Strongly agree No. (%)**	**Number respond-ing**
Overall, I feel that I helped students with their mathematics	0 (0.0)	1 (3.5)	3 (10.6)	18 (63.9)	9 (31.9)	31
I was able to build a relationship with the students	0 (0.0)	2 (6.5)	7 (22.6)	16 (51.6)	6 (19.4)	31
I was welcome in the school	2 (6.3)	0 (0.0)	3 (9.4)	8 (25.0)	19 (59.4)	32
The teacher(s) appreci-ated the help I gave	1 (3.3)	1 (3.3)	3 (10.0)	13 (43.3)	12 (40.0)	30
My role was clearly understood by the school / organisation	1 (3.2)	3 (9.7)	6 (19.4)	10 (32.3)	11 (35.5)	31
I enjoyed the experience overall	2 (6.3)	1 (3.1)	0 (0.0)	9 (28.1)	20 (62.5)	32

With regard to their perceptions of teaching and teachers, as distinct from their own classroom experience, again the responses in general recorded agreement - even strong agreement - with the statements in the questionnaire, as shown in Table 4. The statistical analysis did not support the formation of scales, so results are discussed at item level. Everyone at least agreed with the two social utility items, and no-one disagreed with the altruistic statement that teaching is a noble profession. Most were in agreement with the task demand statements; of the 10 responses expressing disagreement, six came from just two respondents (but a conjecture that the overall responses of these two might be negative in tone was not borne out by an examination of their data). The personal utility and task return items were less strongly endorsed.

Table 4. Level of agreement with statements on demands and benefits in teaching (questionnaire 2; N=32)

Item [and intended "subscale" - see note*]	Strongly disagree No. (%)	Disagree No. (%)	Not sure No. (%)	Agree No. (%)	Strongly agree No. (%)
Teaching is hard work [TD]	1 (3.1)	1 (3.1)	2 (6.3)	14 (43.8)	14 (43.8)
Teachers can help improve society [SU]	0 (0.0)	0 (0.0)	0 (0.0)	8 (25.0)	24 (75.0)
Teaching provides a reliable income [PU]	0 (0.0)	2 (6.3)	16 (50.0)	9 (28.1)	5 (15.6)
Teachers have a respectable social status [TR]	0 (0.0)	4 (12.5)	9 (28.1)	16 (50.0)	3 (9.4)
Teaching is emotionally demanding [TD]	1 (3.1)	1 (3.1)	4 (12.5)	12 (37.5)	14 (43.8)
Teaching is a noble profession [A]	0 (0.0)	0 (0.0)	4 (12.5)	14 (43.8)	14 (43.8)
Teaching offers a steady career path [PU]	0 (0.0)	3 (9.4)	11 (34.4)	11 (34.4)	7 (21.9)
Teachers have a heavy workload [TD]	1 (3.1)	5 (15.6)	6 (18.8)	11 (34.4)	9 (28.1)
Teaching offers good job security [PU]	0 (0.0)	3 (9.4)	7 (21.9)	12 (37.5)	10 (31.3)
Teachers make a worthwhile social contribution [SU]	0 (0.0)	0 (0.0)	0 (0.0)	11 (34.4)	21 (65.6)

Note: TD - Task Demand; TR - Task Return; PU - Personal Utility; SU - Social Utility; A - Altruism

Research question (B): Trends and patterns

Table 5 displays the information on changes in module participants' intention to teach. The first row of data shows the information provided by the 20 respondents to questionnaire 1 who reported taking MA3496, and refers to the module as a whole. The remaining rows display the data from questionnaire 2 and address the classroom experience and its different aspects. Again, some of the latter items were not relevant to some students' experience, so the numbers responding to these items is less than 32 (with percentages calculated accordingly). Overall, interaction with students - perhaps especially with individuals - was reported as causing more change than other aspects listed.

Table 5. Reported effect of module MA3496 and its components on students' intention to teach

Module participants' intention to teach	Less likely No. (%)	Unchanged No. (%)	More likely No. (%)	Number responding
Module overall	2 (10.0)	8 (40.0)	10 (50.0)	20
Classroom experience overall	4 (12.5)	4 (12.5)	24 (75.0)	32
Being back in school as an adult	5 (16.7)	17 (56.7)	8 (26.7)	30
Observing teacher(s) in class	3 (11.1)	11 (40.7)	13 (48.1)	27
Helping students to understand concepts	0 (0.0)	3 (9.7)	28 (90.3)	31
Helping students to develop skills	0 (0.0)	5 (16.1)	26 (83.9)	31
Creating a positive attitude to maths	0 (0.0)	2 (6.5)	29 (93.5)	31
Teaching a topic to a group	2 (8.3)	9 (37.5)	13 (54.2)	24

From the statistical investigations, the comparisons and chi-square tests yielded no significant results. Hence, the focus is on the cross-tabulated open responses. There were 15 comments from questionnaire 1 and 16 of relevance from questionnaire 2; they are represented by 22, quoted in full or with key parts selected, omitted ones typically being near-duplicates or too vague to be helpful. Figures 2, 3 and 4 list comments, respectively from those who stated that they were *more likely* to teach as a result of taking the module (both questionnaires) or specifically the classroom experience (questionnaire 2), *unchanged* in their views, or *less likely*. In each figure, comments are grouped according to whether respondents declared themselves to be *positive* towards going into teaching (answering "Yes" in questionnaire 1 with regard to considering teaching, or in questionnaire 2 agreeing or strongly agreeing that they might teach at some stage), or *unsure*, or *negative* towards it. Comments that refer to teaching in general, rather than to the respondents' experience of the module, are shown in italics. The largest number of comments came from respondents who were more likely than heretofore to consider teaching, although some still classified themselves as unsure or negative about the idea (Figure 2).

***More likely* and *positive* about going into teaching:**

(1) … would now seriously consider it as a career path … felt as if I was truly helping these kids … rewarding [questionnaire 2 (Q2)]
(2) Seeing the fruits of the labour definitely contributed towards a "more likely" intention [Q2]
(3) I had a very hands-on teaching experience .... I doubt I would have enjoyed the experience as much if my role had been, for the most part, observational [Q2]
(4) I learned from the classroom experience that I enjoy working with individual students or with small groups, but that I find it much less appealing to address an entire class [Q2]
(5) … offered an insight into the different aspects and challenges of teaching students of different abilities [Q1]
(6) ... so much more to teaching and mathematics education than I thought, and the theory truly fascinates me [Q1]
(7) The Mathematics Education module has given me … cause to consider not only the possibility of secondary level teaching, but perhaps further study or research in education [Q2]

***More likely* but *unsure* about going into teaching:**

(8) … observing the lack of basic understanding some students had, inspired me to consider teaching, to better equip students … [Q2]
(9) It made me think more critically about how one should go about teaching and how important basic mathematical literacy is [Q1]
(10) It gave a good insight into teaching in schools [Q1]
(11) *Surely a graduate from a mathematical background only needs brief training.... The duration and nature of the masters programme is the main deterrent from training to be teachers* [Q2]
(12) *For newly trained teachers job security isn't always there, particularly when schools need to make cuts* [Q2]

***More likely* but *negative* about going into teaching:**

(13) ... more likely ... because I feel like I have a better understanding of how to teach [Q1]

Figure 2. Responses from students reporting that they were *more likely* to go into teaching

***Unchanged* and *positive* about going into teaching:**

(14) *With regard to social status I feel teachers do not get nearly enough respect in modern society* [Q2]

***Unchanged* and *unsure* about going into teaching:**

(15) it was rewarding and interesting but hard work [Q1]
(16) … interactions with students… were quite limited, and that is why I have answered "unchanged" …. Observing the teachers made me more likely… [Q2]

***Unchanged* and *negative* about going into teaching:**

(17) I did the module to make my workload more manageable [Q1]

Figure 3. Responses from students reporting *unchanged views* about going into teaching

Less likely **and** ***unsure*** **about going into teaching:**
(18) … found supervising classrooms more difficult than anticipated [Q1]
(19) I don't like … day-to-day interactions … with people who are a lot younger than I am. I'm also averse to constantly cycling through the same material year after year [Q2]

Less likely **and** ***negative*** **about going into teaching:**
(20) Even though the classroom experience made me even less likely to become a maths teacher … *it actually affected me quite a lot and made me really sad about educational inequality....* I think the whole experience will stay with me for a while, and I hope to keep doing maths tutoring with disadvantaged students [Q2]
(21) … really enjoyed helping individual students but the incredibly disruptive nature of some students/classes meant I … would not have liked to be in [the teacher's] place [Q2]
(22) The placement put me off teaching [Q1]

Figure 4. Responses from students reporting that they were *less likely* to go into teaching

It was noted above that most respondents to questionnaire 2 experienced regular school classrooms with at least some interactions with students. Interacting with and helping students emerges as an important factor: most clearly positive in comments (1) to (4), (20) and (21). However, some interactions were negative, as indicated in comments (18), (19) and again (4) and (21), dealing with groups being a typical problematic issue. Overall, the comments echo the findings about aspects most likely to predispose students towards teaching, shown in Table 5. They also echo respondents' claims to have helped students and enjoyed their classroom experience, as reported in Table 3.

The specific relationship with mathematics is highlighted in two rather similar comments referring to concerns about understanding (8) and basic literacy (9). However, the focus appears to be on the students again, rather than the subject. Respondents making comments did not explicitly mention, say, the joy of sharing mathematical ideas, though the strong endorsement of the statement about creating a positive attitude to mathematics making them more likely to teach (Table 5) may refer to such aspects. With regard to the respondents' personal views on educational theory, two of the comments - (6) and (7) - single out the study of mathematics education during the module as positive influences.

The data are insufficient to support identification of typologies such as those found by Moses et al. (2017) and Thomson et al. (2012). However, several comments can be tentatively aligned with factors in the FIT-choice model (Figure 1), particularly ones for which items were included in questionnaire 2 (see Table 4); it is of course possible that the items prompted the comments from respondents to that questionnaire. For *Task Perceptions*, comments such as (15), (18) and (21) reflect aspects of Task Demand, while comment (14) indicates a perception of poor Task Return. For *Perceived Values*, many comments on interacting with and helping students, together with (8) and (9) on students' understanding and (20) on educational inequality, appear to reflect Social Utility concerns or perhaps Intrinsic Value. A Personal Utility matter, job security, is the subject of (12).

The latter comment raises an issue germane to the Irish context; despite the shortage of mathematics teachers, permanent positions may not be readily available. Another Irish issue is the increased length of the teacher education programmes. In contrast to (6) and (7), comment (11) appears strongly negative about the academic or professional study of education. Perhaps the insufficiency of subject knowledge for good teaching should be more fully addressed in the lecture component of the module.

While it is hoped that the module will attract students into teaching, there is value also in some students realising that it would not be their correct career choice. Comments (18) to (22), in their different ways, perhaps exemplify this point. Finally, comment (17), which probably refers to the fact that MA3496 is the only mathematics module assessed entirely by coursework, is included as a reminder that students have their own agendas.

Conclusion and implications for teacher education

This paper presents a case study of the effects of a module offered to undergraduate mathematicians in the authors' university as part of their academic programme, providing them with lectures on mathematics education and experience of placements in school classrooms or other such settings. For the classroom experience, typically the undergraduates helped individual students or small groups - for example during "seat work" periods in lessons - though some had only limited opportunities to do so. Two questionnaires were designed for module participants (and the first one also for other undergraduate mathematicians). Overall response rates of 40-50% obviate generalisation to all module participants, but the total of 52 responses from a population of 79 recent participants indicated that the majority of respondents enjoyed their classroom experience, held broadly positive views of teachers and teaching, and did not rule out teaching at some stage of their careers. Most respondents indicated that the module, and specifically the classroom experience, increased the likelihood of their teaching at some stage; a few reported being less likely to teach, but if they were unsuited by or to teaching then this was actually a useful outcome. A key feature in changing respondents' views appeared to be the extent to which they were able to interact substantially with and to help students, as intended but not always achieved by the way in which placements were implemented in schools. Other aspects of the organisational structure - such as whether or not the respondents arranged their own placements and whether it was in a setting other than a regular classroom - did not emerge as explanatory variables, in the latter case perhaps because the number of respondents in other settings was small.

Limitations in the study must be acknowledged. As it is a case study, results cannot be assumed to extend to other groups and contexts. The numbers involved are not large, and the findings are based on self-report via quite short questionnaires, not allowing for the depth of investigation that might be obtained from interviews. So far, only informal feedback has been received on whether the approach is helpful for schools - its prime motivation - and with regard to the actual uptake of teaching by former students. However, the outcomes are sufficiently positive for provision of the module to continue, incorporating improvements where relevant. There are tentative plans to place some students in schools involved in a project run by the university, focusing on an innovative approach to teaching and learning (Lawlor, Conneely, Oldham, Marshall, & Tangney, 2018) - hence, ideally, supporting the teachers' pioneering work as well as giving the students a very rich experience. Also, the possibility of replicating the approach in other subject areas within the university, such as science (with students helping for example during laboratory-based sessions in schools), is being explored.

At a time at which new initiatives are needed in order to attract recruits to teaching, in Ireland and elsewhere, an initiative that makes even a small contribution may be worth exploring. A key feature of the approach described here is that the module forms part of the students' academic study for their mathematics degree, and does not belong to any formal teacher education programme. This allows for freedom in its design and execution. Crucially, it does not have to meet teacher accreditation criteria, and so can focus on flexible implementation that suits the participating university staff, schools and teachers. Undergraduate participants who consolidate or develop an interest in teaching together with realistic expectations about what it entails, based on their classroom experience, may be well placed to progress to formal teacher education programmes. Overall, therefore, the concept may be sufficiently adaptable to be useful in other settings and even perhaps other education systems, encouraging suitable candidates to consider a career in teaching.

Acknowledgements

Sadly, the second author - Donal O'Donovan - died in Autumn 2019. The other two authors would like to pay tribute to his memory and his work, especially here for instigating the module described in this paper.

References

Bergmark, U., Lundström, S., Manderstedt, L., & Palo, A. (2018). Why become a teacher? Student teachers' perceptions of the teaching profession and motives for career choice. *European Journal of Teacher Education, 41*(3), 266-281.

Colhoun, M. (2018). *An investigation into teacher knowledge in the area of mathematics* (Unpublished final year undergraduate project). Trinity College Dublin, the University of Dublin, Dublin, Ireland.

Han, J., & Yin, H. (2016).Teacher motivation: Definition, research development and implications for teachers. *Cogent Education, 3*(1).

Hyland, A. (2019, April 25). Why we are running out of secondary school teachers. *The Irish Times*, p. 5.

Lawlor, J., Conneely, C., Oldham, E., Marshall, K., & Tangney, B. (2018). Bridge21: Teamwork, technology and learning. A pragmatic model for effective 21C team-based learning. *Technology, Pedagogy and Education, 27*(2), 211-232.

Moses, I., Berry, A., Saab, N., & Admiraal, W. (2017). Who wants to become a teacher? Typology of student-teachers' commitment to teaching. *Journal of Education for Teaching*, *43*(4,) 444-457.

Nesje, K., Brandmo, C., & Berger, J.-L. (2018). Motivation to become a teacher: A Norwegian validation of the factors influencing teaching choice scale. *Scandinavian Journal of Educational Research*, *62*(6), 813-831.

Ni Dhuinn, M., Prendergast, M., & Loxley, A. (2019, April*). "Minister can you spare a euro?" The financial and emotional costs of becoming a teacher.* Paper presented at the 2019 annual meeting of the American Educational Research Association. Toronto, Canada.

O'Doherty, T., & Harford, J. (2018). Teacher recruitment: Reflections from Ireland on the current crisis in teacher supply. *European Journal of Teacher Education*, *41*(5), 654-669.

Oldham, E., Colhoun, M., & O'Donovan, D. (2019, August). The influence of an undergraduate module including classroom experience on perceptions of teaching and intention to take up teaching as a career: A study of prospective teachers of mathematics. *Proceedings of the 43rd Annual Conference of the Association for Teacher Education in Europe*, 18-32.

Struyven, K., Jacobs, K., & Dochy, F. (2013). Why do they want to teach? The multiple reasons of different groups of students for undertaking teacher education. *European Journal of Psychology of Education*, *28*(3), 1007-1022.

Thomson, M., Turner, J., & Nietfeld, J. (2012). A typological approach to investigate the teaching career decision: Motivations and beliefs about teaching of prospective teacher candidates. *Teaching and Teacher Education, 28*(3), 324-335.

Watt, H., & Richardson, P. (2007). Factors influencing teaching as a career choice: Development and validation of the FIT-choice scale. *Journal of Experimental Education, 75*(3), 167-202.

Watt, H., Richardson, P., Klusmann, U., Kunter, M., Beyer, B., Trautwein, U., & Baumert, J. (2012). Motivations for choosing teaching as a career: An international comparison using the FIT-choice scale. *Teaching and Teacher Education*, *28*(6), 791-805.

Pre-service teachers' knowledge: Impact on the integration of mathematical applications on the teaching of mathematics

Helena Rocha
UIED, Faculdade de Ciências e Tecnologia, Universidade NOVA de Lisboa, Portugal
hcr@fct.unl.pt

Abstract: Mathematics is present everywhere. However, uncovering the relevance of Mathematics requires, from the teachers, a special kind of knowledge. This study tries to characterize the knowledge used by pre-service teachers when developing a mathematical task intending to promote the students' exploration of barcodes. The study adopts a qualitative and interpretative methodology and the data were collected using class observation and interviews. The analysis is guided by the Application and Pedagogical Content Knowledge, a model inspired on TPACK (from Mishra and Koehler) and MKT (from Ball and colleagues). The conclusions point to some difficulties to see the potential of the situation to promote mathematical learning. The knowledge on the mathematical content seems to be dominant on the options assumed and operated in a rigid way that prevent the pre-service teachers from exploring the richness of the situation on the tasks they developed.

Keywords: applications; mathematics; pre-service teachers; teachers' knowledge

Introduction

Mathematics is present everywhere in our society. Most of what we use nowadays has mathematics on its base. Nevertheless, the interest of the students for its study has been reducing, to the point that it can cause a decrease on the continuous technological development of our society (Osborne & Dillen, 2008). This might be the result of the way how the role of mathematics is usually hidden from the eyes of the users. In these circumstances, uncovering the relevance of mathematics becomes an important and urgent task, and it requires, from the teachers, the ability to propose to the students situations where they can see the presence of mathematics in the reality. However, being able to analyse a situation from reality, identifying the school mathematics on it and developing a mathematical task is not easy. It requires a special kind of knowledge from the teacher.

This study tries to characterize the knowledge used by pre-service teachers when developing a mathematical task intending to promote the students' exploration of a situation from reality. Specifically, it is intended to answer the following research questions: What is the focus of pre-service teachers on the process of designing a task?; What is the role of the different domains of the pre-service teachers' knowledge on the options assumed?.

Teachers' knowledge

Several authors have developed models of teachers' knowledge, identifying and characterizing different domains that integrate that knowledge (Ruthven, 2011). As a consequence, there are several characterizations of teacher professional knowledge developed over the years but, whether we base ourselves on the work of Shulman (1986) or on some other more recent work, all characterizations emphasize the importance of different types of teachers' knowledge.

Mathematics Knowledge for Teaching (MKT)

Hill and Ball (2009) draw on Shulman's work and conceptualized Mathematical Knowledge for Teaching (MKT), where they consider two major areas: SMK - Subject Matter Knowledge and PCK - Pedagogical Content Knowledge (see Figure 1). In the scope of the first, they consider CCK - Common Content Knowledge, SCK - Specialized Content Knowledge and KMH - Knowledge at the Mathematical Horizon. Moreover, in the context of the second, they consider KCS - Knowledge of Content and Students, KCT - Knowledge of Content and Teaching and KC - Knowledge of Curriculum.

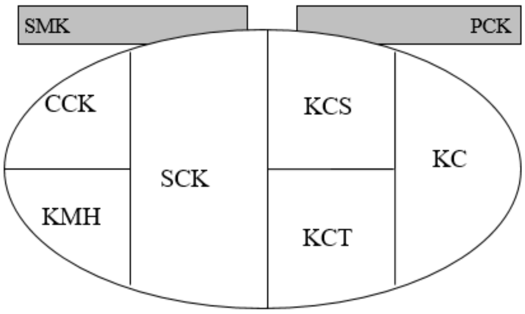


Figure 1. MKT – Mathematical Knowledge for Teaching (Hill & Ball, 2009)

In SMK, Ball, Thames, and Phelps (2008) consider Common Content Knowledge (CCK), which is identical to the knowledge used in other professions where mathematical knowledge is required. It includes knowledge of the definition of a concept or object, how to perform a procedure and how to solve mathematical tasks correctly and even identify the correctness of an answer given by others. The authors also consider a teachers' specific knowledge, the Specialized Content Knowledge (SCK). This is a core knowledge in this model and corresponds to a knowledge that is used in the classroom and that is necessary for the teacher to be able to teach efficiently. It corresponds, for example, to the knowledge needed to identify the source of some error. A third type of knowledge was introduced later by Hill and Ball (2009), the Knowledge at the Mathematical Horizon (KMH). It corresponds to what they describe as a kind of peripheral vision needed to teach. This knowledge encompasses a broad and comprehensive overview of the mathematics teaching landscape, including an awareness of the mathematical topics covered in previous years and how they relate to those approached at the present and in the future.

In addition to these domains of knowledge, Ball, Thames, and Phelps (2008), and Hill and Ball (2009) also refer to a certain integration of content knowledge with another type of knowledge. They thus refer to three domains that we approach here in a very brief way: Knowledge of Content and Student, Knowledge of Content and Teaching, and Knowledge of Content and Curriculum.

Knowledge of Content and Students (KCS) combines knowledge of students and mathematics. This knowledge corresponds to the ability to anticipate students' difficulties, respond to their thoughts and respond to them in a timely and convenient manner. It also includes making an appropriate choice of examples and representations to teach. Both in the preparation process and in the course of its implementation, the teacher must be aware of the conceptions held by the students regarding the topic under study and, in particular, regarding the students' misconceptions.

Knowledge of Content and Teaching (KCT) articulates knowledge about mathematics and teaching. It refers, among other elements, to the teacher's decisions regarding the sequence of activities, his awareness of the possible advantages and disadvantages of the representations used for teaching, his decisions about when to interrupt a discussion in class to clarify some aspects or to use a student's opinion, and also about how to call attention to a certain mathematical aspect.

Teachers' knowledge also includes knowledge of the curriculum, as well as how the different contents interrelate and evolve throughout the school year syllabus.

One of the main points of this model is the way how it emphasizes the mutual influence among different domains of the knowledge. That is, for instance, how the teachers' knowledge of the students impacts the mathematical content and how the teaching approach also impacts the mathematical content.

From TPACK to Application and Pedagogical Content Knowledge (APCK)

The integration of something new on the teachers' practice has proved to be challenging and to require some change on the professional knowledge. One of the most studied situations is the integration of technology. In this case, the need for a different knowledge is deeply recognized and the starting point for the development of some models intending to characterize the teachers' knowledge required to integrate it. One of the most well known models in these circumstances is the TPACK from Mishra and Koehler (2006).

Mishra and Koehler (2006) argue that the articulation of technology knowledge with other types of knowledge is fundamental. According to the authors, the relationships between content, pedagogy and technology are complex and take multiple forms. Indeed, on the one hand, technology has its own imperatives that affect the content to be addressed and its representations and, on the other, it interferes with instructional options and other pedagogical decisions. Decision making regarding the use of technology thus has implications for other areas and as such it does not seem appropriate to consider it in isolation from pedagogical knowledge and content knowledge. Mishra and Koehler (2006) then propose a model that not only considers the three referred domains of knowledge (basic knowledge), but also addresses the connections, interactions and constraints that are established between them. Thus, they consider a Technological Pedagogical and Content Knowledge (TPACK), which is based on content knowledge, pedagogical knowledge and technological knowledge, but also respond to the influences of each of the basic knowledge on each other. They thus refer to Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK) and Pedagogical Content Knowledge (PCK). These three areas of knowledge are the essence of this model and what truly distinguishes it from others previously proposed.

Content Knowledge (CK), Pedagogical Knowledge (PK) and Pedagogical Content Knowledge (PCK) are, taking into account the origin of the model, consistent with the respective notions presented by Shulman and well documented in the literature.

Technology Knowledge (TK) involves the capabilities required to operate a technology and essentially consists of knowing how it works.

Technological Content Knowledge (TCK) is directly linked to how technology and content influence each other. It is a knowledge that, while relying on content knowledge, is different from this. Access to technology not only allows access to different representations, but also facilitates the connection and transition between them. As so, the teacher needs to know not only the content to be taught but also how it can be modified by the use of technology.

Technological Pedagogical Knowledge (TPK) is a knowledge related to the potentialities of technology and the way how teaching can be changed according to the use of it. It includes understanding how a given technology can enhance the accomplishment of a certain type of task, becoming familiar with a set of strategies that allow the students to exploit technology's capabilities, and knowing how to tailor certain teaching methods to integrate technology.

Technological Pedagogical and Content Knowledge (TPACK) is a knowledge developing from the three base components of the model (content knowledge, pedagogy and technology), but goes beyond these. This knowledge is different from that held by a mathematician or a technology expert and equally distinct from the general pedagogical knowledge shared by teachers of different subjects. It is the basis of effective technology integration and requires an understanding of concepts within technology and an understanding of pedagogical techniques that use technology constructively to teach concepts. It also requires a knowledge of what makes a concept difficult or accessible, and how technology can be used to promote students' learning. It also requires a sense of students' prior knowledge and how they learn, as well as a knowledge of how technology can be used to develop existing knowledge or to achieve new knowledge.

Quality teaching thus requires the development of an understanding of the complex relationships between the three base knowledge of the model and the ability to use that understanding to develop an appropriate and context-specific set of strategies.

At TPACK model, besides considering the relevance of the mathematical knowledge, of the pedagogical knowledge and of the technology knowledge, the mutual influence among these domains of knowledge is central. This means that when the technology becomes available, the teacher needs to consider the way it can impact the mathematics and the pedagogical approach. In these circumstances, it seems reasonable to admit that introducing mathematics applications requires some similar development of the teachers' professional knowledge. As so, we propose the model APCK - Application and Pedagogical Content Knowledge (see Figure 2), a conceptualization of the teachers' knowledge similar to the one developed at TPACK.

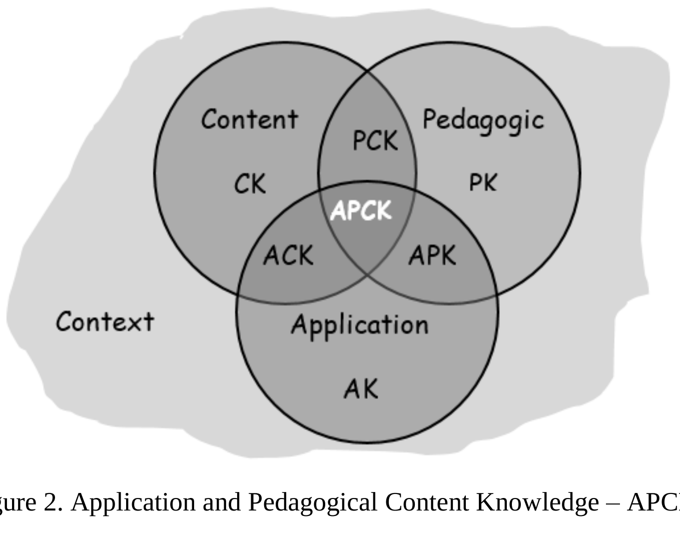


Figure 2. Application and Pedagogical Content Knowledge – APCK

On this model, Application and Content Knowledge (ACK) is directly linked to how the use of Mathematical Applications and the content influence each other. The use of mathematical applications promotes a different use of the mathematical knowledge, where the topics no longer are approached in a specific organized way. When working on mathematical applications all the mathematical knowledge of the students can be used at any time. As a result, the mathematics content no longer is addresses in a compartmented way. On the contrary, all the students' mathematics knowledge can be useful all the time. The consequence is an impact from the use of applications on the mathematical content. In the same way, the Application Pedagogical Knowledge (APK), is directly linked to how the use of Mathematical Applications and the Pedagogical approach influence each other. The use of mathematical applications requires a different approach, where options such as collaborative work, discussion of different approaches and presentation of the work developed are central.

Mathematical applications imply some real context and some interdisciplinary related knowledge. As so, Application Knowledge includes knowledge of disciplines beyond mathematics, and this knowledge is not necessarily mastered by the teacher. The need to look for additional knowledge on fields outside mathematics requires a different attitude, appealing namely to reflection and critical reasoning. Reflection and critical reasoning are important skills for all mathematics' students and can be developed without using mathematical applications. However, applications turn them in central skills. In addition, once again, applications impact how the students learn mathematics (APK) and how the students come to think about what mathematics is (ACK).

From tasks to APCK

On their practice, teachers choose or design tasks, taking decisions about issues such as its characteristics and level of complexity. These decisions are, according to Sullivan and Yang (2013), influenced by their understanding of the relevant mathematics knowledge, by their experience, creativity and access to resources, by their expectations for student engagement, and by their willingness to connect learning to students' reality. And if it is true that tasks do not define the teachers' practice, it is also true that they offer opportunities and limitations (Remillard, 2005).

Mathematical applications are often pointed as having the potential to motivate students, offering them the opportunity to see how mathematics can be useful. However, more important than that, is the potential of these tasks to offer opportunities to live real experiences promoting the students' mathematical learning and understanding (Gravemeijer, 1997). And by a mathematical application we mean an activity set within authentic contexts, and offering opportunities to engage in important mathematical processes, such as describing, analysing, constructing, or reasoning, in line with the idea presented by Ferri and Mousoulides (2017).

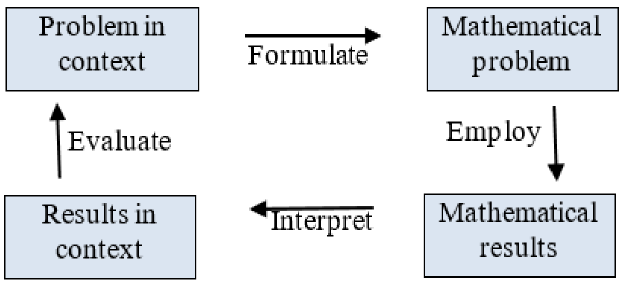


Figure 3. Mathematical application cycle (OECD, 2013)

A mathematical application requires a situation or a problem with a real context. This situation needs to be analysed in order to translate it into a mathematical format. Then, some mathematical knowledge is used to find a result that is interpreted in the context of the situation or problem. This is the cycle (adopted from the framework presented at OECD, 2013) for the work based on a mathematical application (Figure 3).

When designing a task there are several dimensions to consider. Barbosa and Oliveira (2013) speak about arenas of conflicts, mentioning five arenas: *context, language, structure, distribution, subjects*. These conflicts arise from the decisions that the teachers are obliged to make in the process of designing a task. They need to decide, concerning the task *context*, if the task will have a strictly mathematical context, a semi-reality context, or a reality context. Being our tasks mathematical applications, there should be some reality context, however designing the task requires some decisions about it, namely if some simplification of the context will be introduced on the task (case of semi-reality). This requires knowledge about the specific application but also knowledge about the students and the difficulties they might face. As so, this is a decision that requires APK from the teacher. Besides this, the teachers need to decide the level of the *language* used on the task, i.e., if it going to be used some informal language or if the algebraic representation will be required. This implies reflection on the specific application, analysing how it can be translated into mathematical language. It includes issues related to mathematics and to the value ascribed to its specific language (ACK). The teachers also need to take decisions about the task *structure*, i.e., if it going to be a closed-end or an open-ended task. This implies leaving to the students most of the decisions or including some guiding information or questions. In addition, this option requires different pedagogical approaches and the creation of different learning environments in the classroom (APK). The decisions about *distribution*, are the ones related to the content, i.e., to the mathematical focus of the task. The teacher must decide between a task focusing on a single mathematical issue or on more than one. Depending on the structure of the task, the distribution can depend on the students’ options and on the approaches to the task that they decide to follow. This includes issues related to the mathematical content addressed but also to the pedagogical approach, both depending from the fact that we are working with mathematical applications (APCK). At last, the teachers must decide about the *subjects*, i.e., their own role and the students’ role. This is, once again, related to options such as the ones related to the structure of the task (APK).

Methodology and context

In this study, a qualitative and interpretative methodology (Bryman, 2004) was adopted. Data for the two case studies were collected by observation of the pairs of pre-service teachers during two lessons of a course of the master program for pre-service secondary school mathematics teachers and by an interview after the lessons.

Pre-service teachers were presented to a very familiar situation from reality where mathematics plays an important role: the barcodes. Pre-service teachers had the opportunity to get to know the different components of this code, and to learn how to build a barcode, including learning about the process to calculate the control digit (the digit that intends to avoid errors in the reading) of a barcode. After getting familiar with barcodes, the pre-service teachers were invited to develop a mathematical task suitable for their future students, based on this real situation, in order to allow the students exploration of it. The discussions of the pairs of the pre-service teachers during the development of the tasks were then clarified on the interviews. The analysis was guided by the research questions, inferring from the data the categories that guided the pre-service teachers’ options during the development of the task.

Results

In this article, due to space constrains, only the options of the group AB are presented and discussed. This group includes two pre-service teachers, here designated by pre-service teacher A and pre-service teacher B. The task developed by group AB focus on the control digit of the barcode and is presented in Figure 4.

Consider the following barcodes, represented by numbers, and determine the last digit corresponding to the control digit:
560102700121?
505399012374?
560124140400?

And in these cases, can you find the missing digit
56?8653008753
560?722496134
56011?1983508

And if two digits are missing, can you find them?
5601??9007963

Figure 4. Task developed by group AB

As mentioned before, after getting familiar to barcodes, this group developed a task based exclusively on the part of the barcode related to the control digit. Their first idea was to think on a task where the students must get the numbers of the barcode from the view of the black and white bars. However, they state they give up of this idea because they thought the students would not be able to solve it. But they also justify their option based on the mathematics. This group of pre-service teachers was not able to identify the mathematics on the barcodes. The control digit was the part from the barcodes where they manage to identify some mathematical content, and this is an important reason for the characteristics of the task they developed. The conception of this task was driven by a focus on the content and the intension of simplify. In their own words:

> "R: Why did you focus on the control digit? Why did you not include the barcodes?
> B: We began thinking about giving the barcode and ask for the numbers, but… well, we thought it would be very difficult for the students to get the numbers from the bars. And also... we thought it was a task where the focus would not be exactly on mathematics."

Pre-service teachers started reflecting on the *context* and they found it too complex for the students. In these circumstances, they decide to address only part of the real situation: the control digit. However, the *distribution* is also assumed as important on the task, once the pre-service teachers want to focus on some specific mathematical content.

A barcode is a set of 13 digits together with a set of white and black bars (Figure 5). The last digit is the control digit, a digit that intends to detect any error on the reading of the barcode.

Figure 5. Example of the barcode of a Portuguese chocolat

Considering the code as $x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}\ x_{11}\ x_{12}\ x_{13}$, where each x_i represents a digit, the control digit x_{13} is determined so that
$x_1 + 3\ x_2 + x_3 + 3\ x_4 + x_5 + 3\ x_6 + x_7 + 3\ x_8 + x_9 + 3\ x_{10} + x_{11} + 3\ x_{12} + x_{13} \equiv 0 \pmod{10}$.

That is, the thirteenth digit is chosen so that when added to $x_1 + 3\ x_2 + x_3 + 3\ x_4 + x_5 + 3\ x_6 + x_7 + 3\ x_8 + x_9 + 3\ x_{10} + x_{11} + 3\ x_{12}$, the result is a multiple of 10.

Focusing on the control digit, the pre-service teachers identify numbers, and specifically multiples and dividers, as the mathematical content of the task they develop. Nevertheless, pre-service teacher A also mentions problem solving as a content addressed by their task. And this is present on their last question, where there are two digits missing. In this case, it is necessary to reflect on the different possibilities and, as a result, it requires a different type of work than the previous questions.

"R: In terms of content, what will you say is the focus of your task?
A: It's numbers… multiples and dividers…
B: And problem solving."

The options assumed illustrated a focus on the *distribution*, showing the value assigned to the mathematical content. However, it is also possible to identify some concerns relating to the *structure*, reflecting the intention of including problem solving on the task, giving the students the opportunity to analyse and reflect. This intention of including a question more open in its structure, is of course related to the role given to the students and can be seen as a consideration of the *subjects* on the design of the task.

Nevertheless, the focus on the mathematical content is pointed, once again, when the pre-service teachers explain why they decided not to include something on their task focusing on the bars. And the reason, according to them, is that mathematics is not visible on barcodes.

"R: Can´t you think of some mathematical content related to barcodes?
A: I think this kind of situations has not much relation to mathematics."

The translation from a real context situation into a mathematical situation (see Figure 3) is something that does not seem easy for these group of pre-service teachers, a circumstance that will necessarily have implications for the design of the task.

Trying to help the pre-service teachers in the process of identifying mathematics on the barcodes, I suggest the relation between the set of numbers and the bars could be seen as a function. The following excerpt illustrates the difficulty of pre-service teacher A in seeing beyond what can be called a traditional approach to mathematics.

"R: This relation between the numbers and the barcodes couldn´t be seen as a function?
A: A function?... We need an expression for a function.
R: So, you are saying that a function is something like 2x… because in this case we have an expression.
A: Yes.
R: And if I say "the double of a number", instead of 2x, it is no longer a function. Is this what you mean?"

Pre-service teachers seem to have a vision of mathematics too compartmentalized, where the *language* used is also an important issue. This turns difficult for them to move between a real situation and a mathematical situation.

The barcode translates the 13 digits in white and black bars in an interesting way. Looking again at Figure 5, one can see that the first digit appears outside the barcode and that the other numbers are presented in two sets of 6 digits. Each of these three types of digits are then translated into a sequence of 0's and 1's and later on white and black bars (a 0 is represented by a white bar and a 1 by a black bar). On the first set of 6 digits there are two options (the odd and the even one). The decision between these two options is guided by the code in 0's and 1's of the first digit of the barcode (the one outside the bars).

For instance, the digit 5 (first digit of the barcode of Figure 5) is coded as 100110, this means that on the coding of the first set of 6 digits it will be successively used the odd, even, even, odd, odd, and even representation of the digit. This is presented on Figure 6. The translation from the digits to the bars is not addressed in this article, but you can find the codification at the appendix and a more detailed explanation at Rocha and Oitavem (2019). However, you can notice the difference on the bars when we use an odd representation of digit 5 and when we use an even representation of digit 5 (see on Figure 5 and on Figure 6 the difference on the bars for the two 5 digits at the end of the first set of 6 digits). It is possible to consider this odd and even representations as the inverse of the symmetric representation of the other (consider the symmetric image and then change white for black and black for white). To have a clear understanding of this have a look at the appendix and the two columns presented for the first group of 6 digits.

Therefore, barcodes are sets of white and black bars where 12 digits are translated directly into the bars and one of the digits (the first one) is translated indirectly.

6	0	1	0	5	5
odd	even	even	odd	odd	even
1	0	0	1	1	0
5 (1st digit of the barcode)					

Figure 6. Coding of the first set of six digits and of the initial digit

Trying to help the pre-service teachers' exploration of the situation, I challenged them to think about an increase of the number of digits of the barcode from 13 to 14. Thinking about ways of coding the information of an additional digit, led them to think on combinatorial calculus as one mathematical content involved on the situation.

> "R: And if your task asks the students to create a barcode to represent 14 digits instead of 13. (…)
> B: We could consider two digits out of the barcode instead of just one.
> A: Yes, but now instead of 10 cases we have 100. We need to know if it is possible to create 100 different sets of six bars.
> B: Right! So, using 0's and 1's in six positions how many different numbers can we create? Eh, this looks like that part of combinatorial calculus!"

The focus on the identification of the mathematical content present on the situation - *distribution* - seems to be a concern always present, even when the pre-service teachers are trying to think about a challenge that was posed to them.

Reflecting deeper on the situation, the pre-service teachers also identified another mathematical content involved, the symmetry of figures. However, their focus on the content and curriculum raised another question related to the usefulness of the situation:

there is no single level where combinatorial calculus and symmetry of figures is addressed.

> "A: We need to see if there are no repetitions when we consider the symmetric bar and also when we consider… let's say its inverse… when we change the black bars to white bars and the opposite… I think we can face problems here…
> B: So, we have to work both, combinatorial calculus and symmetry of figures… but we don't have both at any single level…
> A: And what about the control digit?... I think you are right. It's not possible, we have to consider mathematical content from different levels…"

Once again, there is a focus on *distribution* impacting the design of the task. But more than identifying the mathematical content addressed, there is a concern to address it in a rigid way, following in a linear way what is prescribed on the curriculum, and not valuing the use of knowledge learned on previous schools years. This suggest that open-ended tasks are not valued, which as impact on the task's *structure* and on the students and teachers' role, i.e., on the options related to the *subjects*.

The use of mathematical content from a previous level is not considered as a good idea to promote new learning. The pre-service teachers think about the students and assume that a task requiring knowledge on content from previous levels will be too demanding for the students. And if reviewing content previously studied can be a possibility, it seems not to be assumed as the most desirable one or the most important one.

> "B: I think that such a task might be too difficult for the students. I mean, if they have to use the mathematics worked on previous years…
> A: Maybe… but somehow that can be good, can't it? It is an opportunity to review content from previous levels."

Globally, the intention seems to rest on the simplification of the *context* to avoid difficulties.

Conclusions

The main conclusions point to some difficulties to see the potential of the situation to promote mathematical learning and to show the beauty and power of mathematics. The task developed by the pre-service teachers tend to be exercises focused exclusively on one mathematical topic. Besides that, ideas that are beyond the school content are neglected. This suggested an appreciation for the *distribution*, also identified by Dawn (2018) that in the case of this study tends to be associated with a simplification of the *context*. The limitation on the mathematical content addressed tend to lead to a task with a closed *structure*, which impacts the roles assumed by the teacher and the students, impacting the *subjects*. The *structure* is also a sensible field identified in other studies (e.g., Dawn, 2018), however this study has the specificity of asking for the design of a task for a given context. In cases where the tasks are developed for several contexts chosen by the teachers, a more diversified *structure* for the tasks was identified. The *language* issues do not seem to be very present in this study, however, somehow, they can be responsible for the difficulties in translating from a real situation to a mathematical situation. Globally, the study shows that exploring a real situation challenges the pre-service teachers' knowledge, in line with the findings of other studies (Blum, 2015). The knowledge on the mathematical content seems to be dominant on the options assumed and operated in a rigid way that prevents the pre-service teachers from exploring the richness of the situation on the tasks they develop. This seems to be the result of some weakness on the pre-service teachers' ACK: pre-service teachers face difficulties to translate from the real situation/application to the mathematical situation. As a consequence, some pedagogic options are compromised (such as open-ended tasks and more active roles to the students) and this seems to prevent the development of their APK, and consequently their APCK.

This conclusion emphasizes the relevance of the professional knowledge as well as the need to a deeper understanding about how to promote its development. Understanding the impact of the professional knowledge on the pre-service teachers' options is central to improve the training programs.

Acknowledgements

This work was supported by the Portuguese Science Foundation - FCT, through the project PTDC/CED-EDG/32422/2017.

References

Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special?. *Journal of Teacher Education*, *59*(5), 389-407.

Barbosa, J., & Oliveira, A. (2013, July, 2013). Conflicts in designing tasks at collaborative groups. *Task design in mathematics education - Proceedings of 22th ICMI Study*, 541-548.

Blum, W. (2015, July 2012). Quality teaching of mathematical modelling: What do we know, what can we do?. *Proceedings of the 12th International Congress on Mathematical Education*, 73-96.

Bryman, A. (2004). *Social research methods*. Oxford: Oxford University Press.

Dawn, NG. (2018). Towards a professional development framework for mathematical modelling: The case of Singapore teachers. *ZDM*, *50*(1-2), 287-300.

Ferri, R., & Mousoulides, N. (2017). Mathematical modelling as a prototype for interdisciplinary mathematics education? - Theoretical reflections. In T. Dooley & G. Gueudet (Eds.), *Proceedings of CERME 10* (pp. 900-907). Dublin, Ireland: ERME.

Gravemeijer, K. (1997). Instructional design for reform in mathematics education. In M. Beishuizen, K. Gravemeijer, & E. van Lieshout (Eds.), *The role of contexts and models in the development of mathematical strategies and procedures* (pp. 13-34). Utrecht, The Netherlands: Technipress.

Hill, H., & Ball, D. (2009). The curious - and crucial - case of mathematical knowledge for teaching. *Phi Delta Kappan*, *91*(2), 68-71.

Mishra, P., & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, *108*(6), 1017-1054.

OECD (2013). *PISA 2012 assessment and analytical framework: Mathematics, reading, science, problem solving and financial literacy*. Paris: OECD Publishing.

Osborne, J., & Dillon, J. (2008). *Science education in Europe: Critical reflections*. London: The Nuffield Foundation.

Remillard, J. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research*, *75*(2), 211-246.

Rocha, H., & Oitavem, I. (2019). Barcodes: The mathematics of everyday life. *The Scottish*

Mathematical Council Journal, 49 (in press).

Ruthven, K. (2011). Conceptualising mathematical knowledge in teaching. In T. Rowland & K. Ruthven (Eds.), *Mathematics knowledge in teaching* (pp. 83-96). Dordrecht: Springer.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.

Sullivan, P., & Yang, Y. (2013, July, 2013). Features of task design informing teachers' decisions. *Task design in mathematics education - Proceedings of 22th ICMI Study*, 529-530.

Appendix

digit	1st digit	1st group of 6-digits		2nd group of 6-digits
0 0	111111	0001101	0100111	1110010
1 1	110100	0011001	0110011	1100110
2 2	110010	0010011	0011011	1101100
3 3	110001	0111101	0100001	1000010
4 4	101100	0100011	0011101	1011100
5 5	100110	0110001	0111001	1001110
6 6	100011	0101111	0000101	1010000
7 7	101010	0111011	0010001	1000100
8 8	101001	0110111	0001001	1001000
9 9	100101	0001011	0010111	1110100
parity		odd	even	

Learning to enhance children's creativity in a makerspace

Monique Pijls, Marco Kragten & Tom van Eijck
Centre for Applied Research in Education (CARE), Amsterdam University of Applied Sciences, The Netherlands
m.h.j.pijls@hva.nl | m.kragten@hva.nl | t.j.w.van.eijck@hva.nl

Abstract: Maker education offers opportunities to stimulate the creativity of young people in various types of education. How to guide these learning processes, however, is an unexplored area for the supervisors (teachers and librarians). In the research-project presented, a professional learning community of librarians, teacher-researchers and maker educators investigates the pedagogy of 'making'. The learning community consisted of twelve makerspace-coaches, three maker educators and three researchers. The interventions for enhancing creativity that were developed varied from redesign of the tasks to new forms of guiding students. It was noticed that the children came up with new ideas and were motivated to push out their frontiers. Furthermore, the coaches experienced that children's creativity is not always visible in the final products of their making process, but rather in the process of making. The learning community turned out to be a fruitful approach for professionalization of makerspace-coaches.

Keywords: creativity; learning community; maker education

Introduction

In the past few years, 'learning by making' is finding its way in both primary and secondary education. Teachers discover new opportunities for inquiry, learning by design and integration with other subjects in maker projects (Martin, 2015). "Making" means an integration of creativity and new technology (3D printer, laser cutter, vinyl cutter, and more) and all possible other tools. This requires space and machines, which is a major investment for many schools. Public libraries see their task here to create makerspaces were school classes can come with their teacher and children after-school-programs (Slatter & Howard, 2013; Caso & Kuijper, 2019). These manufacturing locations can differ from each other on many points (Sheridan et al., 2014). Learning in a maker-space is anchored in the experience of making, with the process of tinkering, sorting out things, playing with materials and tools being of paramount importance. As a creator in these spaces, you need to find problems and projects to work on, join a community, take leadership or take a different role if needed and share creations and skills with a wider world. This requires a culture of trying out, asking questions and helping each other (Bevan, Ryoo, & Shea, 2017).

The present study focuses on the question of how these professionals can professionalize themselves to become "makerspace-coach", in particular with regard to stimulating creativity. We believe that the results are important for teacher educators because the professionalization of teachers into makerspace-coach will play a role in the coming years as schools expand their education with maker education as teaching for creativity is new for many (science) teachers and they lack skills to support children's learning processes in the maker space.

The aim of this study is to answer the following research question: How do makerspace coaches learn to enhance children's creativity in a library makerspace by intervening in tasks, materials and guidance of the coach?.

This study is part of a larger project wherein learning goals in the makerspace - citizenship, empowerment and peer learning - will be explored.

Theoretical framework

Learning in the makerspace can be placed in the three target domains of education (Biesta, 2012), as shown in Table 1. First of all, there is the domain of qualification, i.e., the knowledge, skills and attitudes in the field of new technology and creativity. Children learn in the workplace to research and design and make use of digital manufacturing. Secondly, learning also involves socialization, where children are included in traditions and practices. On the one hand, these are the practices that relate to functioning as a researcher and designer, but also to democratic citizenship, which are involved in the environment. Thirdly, there is the domain of subjectification, of becoming a person in the world. That is about initiative, inspiration, motivation, and confidence in your own abilities.

Table 1. Domains of learning in the makerspace

Domain in the makerspace	**Learning goals**
Qualification - Technology and creativity	2D and 3D design, use tools, computer programming, to investigate, media literacy
Socialisation	Visualization and articulation of ideas, helping others and giving explanations, justifying your own work, being involved in a living environment, citizenship
Subjectification	Taking initiative, identity, self-confidence, self-efficacy, inspiration, enthusiasm, perseverance

Creativity in the workplace allows you to create meaningful objects or ideas in an original and personal way in freedom where the development of 21st century skills is central. Creators can show creativity in an increasing degree. It often starts with 1) learning to use technology (*reproducing*) then 2) introducing your own variation (*playing*) and even 3) trying new things (*experimenting*) up to 4) creating something new (inventing) (Loertscher, Preddy, & Derry, 2013). Characteristic for makers is the ability to improvise and put themselves in the position of future users of what they make (Teng, Chuang, & Hsu, 2015). Guiding children's in their creative process requires both restriction and openness, and room for frustration (De Jong, 2017).

Method

In the research-project presented, a professional learning community of librarians, teacher-researchers and maker educators jointly investigated the pedagogy of making. The learning community consisted of twelve makerspace-coaches, three maker educators and three researchers. The makerspace-coaches had been trained in digital technology and making in the past two years by the maker educators. Two of the researchers were self-taught makers. The makerspace-coaches provided programs in the makerspace for almost 60 children. The learning community focused during six months on the question how to enhance children's creativity.

Five monthly meetings of two-and-a-half hours were arranged in order to collaboratively: 1) Create a common framework of 'creativity in the makerspace' by integrating theory and experience from practice; 2) Develop interventions to enhance creativity in the makerspace; 3) Perform the interventions and collect data; 4) Analyse results and formulate conclusions.

Data-collection consisted of reports of the five meetings, field-notes of the makerspace-coaches, pictures of children's artefacts, learner reports of the coaches and recording of the focus interview. The 30 children involved in this study visited one of the three makerspaces of the public library that was located in their neighbourhood. They followed a 12-week workshop on digital manufacturing. The majority of the children had followed one or more workshop(s) in the makerspace in the past year.

Results

The learning community

In this section, we discuss the results of the meetings of the Learning Community.

First meeting - A common definition of creativity in a makerspace

The coaches started with a brainstorm on the concept 'creativity' and made a mind-map on a large sheet of paper. Secondly, they were presented definitions of creativity from literature on little cards as shown in Table 2.

Table 2. Cards with definitions of creativity that were investigated by the coaches

Creative ability enables you to find new or original and applicable ideas for existing questions and is a combination of **originality** and **functionality**. (Buisman, van Loon-Dikkers, Boogaard, & van Schooten, 2017)	Creative ability = **Divergent thinking** (explorative, aimed at generating new ideas, solutions or alternative) + **Convergent thinking** (focusing on integrating new ideas or alternatives to a new approach or solution) (Buisman, van Loon-Dikkers, Boogaard, & van Schooten, 2017)	Creative thinking reflects the *original interpretation of experience*. Each of us has the capacity to construct original interpretations, and if it is a useful and original interpretation, it qualifies as "creative" (Runco, 2008)
The ability to be creative in a certain domain (science, arts, music, programming) is based on years of study and acquaintance of domain-specific skills (Sawyer, 2012)	Creativity is ' possibility thinking' with seven *habits of mind*: • questioning • play • immersion and making connections • being imaginative • innovation • risk taking • self determination (Cremin, Burnard & Craft, 2006)	

Thereafter, they discussed their mind-maps and reflections on the definitions. This yielded in the following findings.

- *Can creativity be taught? - Creativity cannot be enforced.* When exploring the notion of 'creativity' it first came up that it is hard to steer creativity, that one cannot 'grab' it, that one should not exert pressure, that it is important to give space as much as possible. ‘When it is obligatory, then I get blocked’, told one of the coaches.

- *Creativity demands space to be ‘different’ and to ‘make mistakes’.* Stimulating creativity has to do with generating trust and offer a safe environment. Creativity can be ‘act silly’ where it is important not to be laughed at. Being allowed to fail plays an important role. It was stressed that creativity is more a process than a product.
- *Creativity has both a functional and an artistic side.* On the one hand finding practical solutions, designing life-hacks and on the other hand creating, artistic, non-necessarily functional. One of the coaches mentioned that she entered the meeting with an image of creativity connected to arts, but that she came to the insight that it is linked to technology too.
- *Creativity is a collaborative process.* The coaches mentioned that many definitions of creativity emphasize the development of the individual, but that in a makerspace it is important to create *together*. This has to do with sharing knowledge and skills - an important aspect of the maker mind-set - but also with helping each other to generate ideas, or in a practical way. Another
- *Can creativity be taught? Sometimes it starts with copying.* A question that was raised: do we as makerspace coaches expect that children that come to the afterschool programs are creative or do we teach them to be creative? The experience is that children who are forced to come to the makerspace - this happens sometimes, since - are less easy to motivate and the coaches noticed that these children were mainly copying ideas from others.

The framework for creativity of Buisman et al. (2017) was discussed and adapted to the situation in the makerspace and provided with examples in the weeks that followed. In the end, this yielded in the framework ‘Creativity in a maker space’ as shown in Figure 1.

Figure 1. Re-design of the framework Creativity in a makerspace

The first column shows the five aspects of creativity with sub aspects (second column), and then each aspect is operationalized with behaviour (third column), an example from the maker space (fourth column) and how coaches enhance this children's behaviour (fifth column). As an example, the first aspect, curiosity (i.e. the first three rows of the framework) is shown in Table 3.

Table 3. The aspect 'Curiosity' from the framework Creativity in a Makerspace

Sub aspect	**Behaviour**	**What we see**	**Coach is doing**
Observing	Observes materials, tools and situations	*Child enters and observes creations* *Boy observes 3D-printer*	Draws attention to the environment of the makerspace
Questioning	Asks himself and others questions about making	*Kids ask which other activities take place*	Stimulates questioning
Investigating	Makes a brainstorm Looks beyond the first idea	*Kid searches on Google to ideas and connects this*	Stimulates research

The framework 'Creativity in a makerspace' was printed on an A2-poster that got a place in every makerspace as a reminder for coaches.

Second meeting - Pedagogy of creativity

In the second meeting the coaches gathered ideas on stimulating creativity with children. These ideas from practice were compared with pedagogical approaches from literature. We discerned interventions on three domains:

- *Makerspace environment* (materials, tools …)
  (5 times mentioned by coaches);
  - creating a safe and quite place, playing music
  - offering tools
  - showing the machines
- *Instruction materials* (open & closed tasks, manuals …);
  (7 times mentioned by coaches)
  - giving time for reflection
  - open tasks
  - starters, warming up
- *Guidance of the coach* (scaffolding, supervision …).
  (44 times mentioned by the coaches)
  - showing interest in the children
  - making them feel safe to express themselves
  - becoming friends
  - allowing to fail
  - stimulating ideas
  - stimulating interaction with peers

Third meeting - The interventions

The coaches selected one aspect of creativity from the framework and for each makerspace, a team of coaches developed an intervention to stimulate creativity. Two of the interventions that were developed:

- *Magic lantern: repeating the same laser-cutting task.* One of the coaches signalled that children sometimes rushed through a certain task and then immediately continued with another project, while their creation could be improved. This coach intended to make children do the same task again in order to stimulate convergent and divergent thinking.

- *Superhero: an open programming design task.* The coaches noticed that many children performed instructions individually, but did not apply what they learned in an own design. They wanted to change this by a surprising design task. Aim of the intervention was to stimulate inventiveness during the programming course.
- *Ask questions.* Let children reflect on what they are making, by reflecting on this both at the beginning of the afternoon and by questioning them.

Fourth meeting - Results

- *Magic lantern.* The coaches stimulated the children on purpose to redesign their first artefact. About this, they report:

> "What happened with the kids when we gave them a second chance? The pictures speak for themselves. The children were freer in their second design. They knew better what was expected from them, they became freer in their head and they were less concerned with the process. This made them more creative and self-assured. We made lanterns with them. They were given an explanation about the technology with the projector beforehand and were shown a number of examples".

- *Superhero.* Upon entering the workshop, the children saw a picture of an exploded globe. The assignment for children was "Design a superhero who will save the earth". With the help of photos, the coaches have recorded how the children proceeded. Material for this was glue gun, cardboard, tape and of course Microbit. The coaches have taken photos (see for example Figure 5) of the four groups at work and children all came to an end product. The coaches wrote about this:

> "Based on the end product, it is difficult to establish ingenuity/creativity, we think the process says much more. The ambition of this assignment was quite high in terms of time, each group had an end result, but a few were not ready. More time may be needed to stimulate divergent thinking; to go beyond the initial idea. "

- *Asking questions.* The intention of this intervention was to let children reflect on what they do. When questioning the children, it appeared that children asked for guidance at that time. They wrote the following about this:

> "Children seem to have difficulty coming up with something to make themselves and ask the coaches for specific / directed assignments. Not being able to come up with a design or solution for a problem yourself often leads to frustration and sometimes "disruptive" behaviour. Reflecting is also visibly more difficult if no goal is set in advance. No matter how small that goal is. This has made us think further. Our main question has now become: How do you tap into the (natural) creativity and curiosity of children? What are good activating questions?"

Fifth meeting - Learning experiences of the coaches

The learning experiences of the coaches were inventoried during the last meeting of the design cycle on creativity. The evaluation consisted of two phases: each coach first completed a learner report and then two focus group discussions were held. The central questions during the evaluation were: what have the creative coaches learned about creativity?; what have the job site coaches learned about guiding and promoting creativity in the job site?; what have the job site coaches learned from working in the Learning Community?.

The learner report consisted of nine open questions and was filled in by coaches during the meeting. Subsequently, two focus group discussions were held with the coaches simultaneously. Focus group 1 consisted of 6 coaches and focus group 2 consisted of 8 coaches. Each conversation lasted approximately 45 minutes. During the focus group discussions, the questions from the learner report were used as a guideline. These allowed coaches to further clarify the answers and discuss similarities and differences in learning experiences. The answers of the coaches are transcribed and analysed by horizontal comparison.

The coaches indicated that the different theoretical perspectives offered taught them that creativity can be defined in different ways. It appears to be a container concept and there is always some subjectivity, but that many coaches considered it to be measurable as an eye-opener. Several coaches also indicated that they are now more aware that creativity is something that you can learn and stimulate instead of a quality that you do or do not have. By formulating a definition of creativity in the workplace with all coaches, it was jointly experienced that the process is more important than the end product and that it is not only about making physical things but also about creating ideas. The coaches had also developed a framework for creativity in the workplace with concrete examples of creative behaviour. Most coaches perceive this framework as a good hold and it helps them to pay attention to specific behaviours in children that they would perhaps rather be less aware of. In addition, it was stated several times that the framework helped to structure the supervision. It should be noted that the framework was not always sufficiently applied in practice.

> "…all the theory and knowledge about it has helped me to recognize creativity earlier ..."

> "... I then started paying attention to things I hadn't thought of..."

Most coaches indicate that designing an intervention to promote creativity helped them to be more aware of their role and potential. They felt more capable of dealing with the different initial situations of the children and guiding them through a creative process. The joint design was experienced as very instructive. The implementation of the intervention and the associated data collection had also yielded several learning experiences. Several coaches pointed out that they had become more aware of the importance of a well-defined assignment. Too much freedom and openness can sometimes have a paralyzing effect on children because they are usually too inexperienced to get started themselves. Creativity is a process that you have to guide as a coach and most coaches indicate that showing examples is important here. In addition, it was clearly experienced that it is necessary for both coaches and children to have time to be able to iterate.

> "... time is very important, there was also a second group who took a lot of time and did it multiple times, so there was time to try it out and be divergent, to think up different things, to pick something out or add something. That is what we have learned: two hours is just too short, we were too ambitious, that is what we gradually found out... "

> "…We have learned to put it together, 2 hours is just too short, too much hay on the fork, you will gradually find out... "

The learning experiences of coaches with regard to guiding and promoting creativity in the workplace were largely the same. It was often mentioned that it is important to let children talk about what they are making and to ask the right questions. What is 'right' to ask differs in time and per person. The framework 'Creativity in a Makerspace' shows the various aspects of the creative process and how to guide them. In the beginning of the process, when a child has just entered the makerspace, it might be appropriate to raise curiosity and ask for children's experiences and wishes. In another phase it might be better talk about technical details. Consciousness of these aspects helped the coaches to formulate the 'right' questions. In addition, it was often mentioned that it is important to be positive, to motivate, to give good examples and to be a good example. Daring to let go of the planning was also often mentioned as an important learning experience. Dealing with children with specific problems, such as autism, was still experienced as difficult by a number of coaches.

> "… Asking the right questions, it was very useful for me to have those questions and to be aware of what do you want the child to learn and how and with the right question you can ask. That was really a Eureka moment..."

Most coaches indicated that they had learned a lot from the meetings of the learning community. The structure and structure of the meetings was clear and the personal contribution and discussions with each other are inspiring and make it very relevant. A few coaches indicated that they find this form of professionalization a clear addition to the existing offering because the emphasis is more on the questions about the "why and what" of the place of manufacture. The development of a joint professional language and vision was also mentioned as an important learning experience. It should be noted that it was an intensive way of learning that takes a lot of time, making it sometimes difficult to combine with other activities. In addition, there was a great diversity of views and experiences in the group, so that there are sometimes insufficient opportunities to delve into each other's situation and positions.

> "... lots of material to think about and become aware of and really practical tools. And also hear how others have that, otherwise you will have very little at that moment, then it will really be broadened, very pleasant ..."

Conclusions

The design research focused on the question how makerspace coaches can enhance the creativity in a maker space. This question was investigated in a learning community with maker space coaches and researchers. First, a framework for creativity in a maker space was developed. Secondly, the coaches developed interventions to enhance creativity.

They learned that creativity in a maker space:

- consists of in *many aspects*: observing (curiosity), questioning, investigating, making connections (inventiveness), convergent and divergent thinking, daring, creating, re-designing, reflecting, sharing;
- often becomes visible in the *process* of making than in the products children make;
- *can be enhanced* by the makerspace coaches. Creativity appeared to be measurable for the coaches and they experienced how to stimulate it. This was an eye-opener, since some coaches perceived it as a 'gift'. They became curious to get to know more of the learning process of the children in this aspect. Developing creativity and learning is important for children in developing their technological literacy.

(Future) teachers need to professionalize in enhancing children's creativity in their lessons.

The makerspace coaches experienced that collaborative design and investigations in the Learning Community helped them to learn how to better help children. The approach consisted of 1) creating a common ground with definitions of creativity from theory and experience in practice 2) checking the framework in practice and refining 3) developing interventions to enhance creativity 4) performing the interventions and collecting data; 5) analysing results and formulating conclusions. Step 4), however, which consisted of the process of data-collection by the coaches, was hard to combine with the organization of and responsibility for the program in the maker space. That is why, in the continuation of the project, researchers will mainly perform the data-collection.

What does this project teach us about the professionalization of teachers? Guiding creative processes requires a specific approach, which is new to most teachers. Going through the design cycle together in a learning community - as described above - appears to be a suitable way to learn this. In addition, it is important to note that prior to this study, the production makerspace-coaches at the library were trained in the field of digital technology and making, so they were trained as makers themselves. Both elements, self-making and learning to supervise the making of children are important for (prospective) teachers to make use of the makerspace in their education.

Acknowledgements

This research was funded by the municipality of Amsterdam as part of the project Maakplaats 021, carried out by the Amsterdam Central Library (ObA), Waag Society, Pakhuis de Zwijger and the Amsterdam University of Applied Sciences.

References

Bevan, B., Ryoo, J., & Shea, M. (2017). What if? Building creative cultures for STEM making and learning. *Afterschool Matters, 25*, 1-8.

Biesta, G. (2012). *Goed onderwijs en de cultuur van het meten*. Den Haag: Boom/Lemma.

Buisman, M., van Loon-Dikkers, L., Boogaard, M., & van Schooten, E. (2017). *Stimuleren van creatief vermogen en kritisch denken.* Amsterdam: Kohnstamm Instituut.

Caso, O., & Kuijper, J. (2019). *Atlas. Makerspaces in public libraries in The Netherlands.* Delft: TU Delft Open, Faculty of Architecture and the Built Environment.

Cremin, T., Burnard, P., & Craft, A. (2006). Pedagogy and possibility thinking in the early years. *International Journal of Thinking Skills and Creativity, 1*(2), 108-119.

De Jong, L. (2017). Het begeleiden van creatieve processen. *Cultuur+Educatie, 16*(47), 62-84.

Loertscher, D., Preddy, L., & Derry, B. (2013). Makerspaces in the school library learning commons and the uTEC maker model. *Teacher Librarian, 41*(2), 48-67.

Martin, L. (2015). The promise of the maker movement for education. *The Journal of Pre-College Engineering Research, 5*(1), 30-39.

Pijls, M., van Eijck, T., & Kragten, M. (2019, April). *Learning to enhance creativity in a maker space*. Paper presented at the Winter Conference of the Association of Teacher Educators in Europe. Braga, Portugal. (See in these Proceedings).

Sawyer, R.K. (2012). *Explaining Creativity: The Science of Human Innovation*. Oxford: Oxford University Press.

Sheridan, K., Rosenfeld Halverson, E., Litts, B., Grahms, L., Jacobs-Priebe, L., & Owens, R. (2014). Learning in the making: A comparative case study of three makerspaces. *Harvard Educational Review, 84*(4), 505-531.

Slatter, C., & Howard, Z. (2013). A place to make, hack and learn: Makerspaces in Australian public libraries. *The Australian Library Journal, 62*(4), 272-284.

Teng, C., Chuang, M., & Hsu, C. (2015). Planning a design course for play experience and fablab. *International Journal of Educational Research, 3*(10), 751-756.

How do future preschool teachers perceive practical work for their initial and future training?

Lourdes Aragón
Universidad de Cádiz, España
lourdes.aragon@uca.es

Abstract: This paper analyses the perception of future preschool teachers towards the application of practical work in order to enhance their interest and positive attitude towards science. This study was undertaken within a teaching innovative practice in the topic 'Didáctica del Medio Natural' at the University of Cádiz (Spain) of the 3rd Early Childhood Degree. The opinion of students was analysed based on three aspects: 1) Usefulness for professional training; (2) Degree of satisfaction with contents; and (3) Degree of utility for your future teaching career. For this project, quantitative analysis was designed using three-closed questions-Likert type. Each question was established having four different levels of agreement or disagreement as possible response. Results indicate that students were very satisfied with the development of practical work. It is expected that this research can contribute to improve the interest of teachers in science.

Keywords: environmental science; perceptions; practical work; teacher initial training

Introduction

Some authors (such as Grilli-Silva (2018)) argued that teaching and learning science aims at the introduction of students into the scientific method used to build its own knowledge. In order to achieve this purpose, practical work appears to be the most adequate activity allowing to cover a wide variety of objectives (Caamaño, 2003). The term 'practical work' is used in science to refer to certain activities which require students to perform diverse actions to solve them and that encompass laboratory activities and fieldwork (Del Carmen, 2000). Caamaño (2003) argues that practical works can be very diverse. They range from experiences, that make the student familiar with the case of study, to illustrative experiments, that allow us to show the relationship between variables and practical skills. All these activities help in the learning process of new skills which are relevant in science research contexts, giving the students the opportunity to play the role of scientists.

Literature demonstrates that, in general, future teachers and current teachers, value practical activities positively, mainly because they allow to relate theory with practice. However, they do not seem to consider so much the impact of this type of activities to cover conceptual learning or as an introduction to scientific work (García Barros, Martínez Losada & Mondelo Alonso, 1998). On the other hand, nowadays, the affective component and attitude are considered as factors that play a pivotal role of learning and teaching science. According to Mellado et al. (2014), negative emotions can act as real barriers students, as well as for educators (Santmartí, 2001) in a learning process. In this sense, future teachers show negative attitudes and emotions towards teaching science, considering it boring, difficult and far from daily life. Several studies (Mazas & Bravo, 2018; Ocaña, Quijano, & Toribio, 2013) reveal a poor attitude by future teachers towards the learning of science.

Furthermore, it is not only the lack of interest from teachers regarding science, but also the fact that, as shown by a number of research studies, current teachers have a low-profile in science (Abell, 2007; Murphy, Neil, & Beggs, 2007). This fact appears as one of the main obstacles when it comes to addressing science at school (Cantó & Serrano, 2017). Besides, studies make evident the low presence, in preparatory school, of science teaching activities focused on the scientific methodology or the argumentation (Cantó, de Pro, & Solbes, 2016). Having this into account, teachers who dedicate themselves to initial teacher training, have a great challenge ahead. For this reason, it is necessary to to design educational proposals able to foster the interest of our students in science while favouring the development of positive attitudes towards this subject. Practical work is proposed as an adequate strategy for increasing motivation, stimulation and development of scientific attitudes in students, among other skills (Hodson, 1992; Séré, 2002).

Research aim

During several academic courses, academic staff in the Department of Didactics and Experimental Sciences of the Faculty of Education Sciences of the University of Cádiz (Spain) have observed that students have low profile and adverse attitudes towards science, considering this school subject difficult and tedious. In this context, during the academic year 2018-2019 an innovative proposal was designed. Four practical sessions were designed under the topic Didáctica del Medio Natural (DMN), based on both laboratory and field trip. Practical work was implemented in order to increase the interest and to develop positive students' attitudes towards science. The main research question in this study is: How do future preschool teachers perceive practical work for their initial and future training?. Therefore, the paper aimed at analysing students' perceptions after four practical sessions. In this research, the opinion from students was analysed based on three aspects: 1) Usefulness for professional training; (2) Degree of satisfaction with contents; and (3) Degree of utility for the future teaching profession.

Research methodology

Population and sample

This research focused on the topic Didáctica del Medio Natural, at the University of Cadiz (Spain), in the 3rd year of a degree programme leading to early childhood education. The topic is mandatory and it is the only one covering experimental sciences teaching during the degree leading programme. The topic covers the first semester and has a duration of 15 weeks. The practical works were designed and distributed in four sessions. Each session has one hour and a half of duration and it includes individual activities, alternating between small groups and large groups. Thus, students were normally grouped into work teams formed by 5-6 members each. A total of 56 students, 7 men and 49 women, aged between 19 and 41 years, have participated in the study.

Teaching intervention

The practical works were implemented simultaneously in tutorial sessions. The contents that were introduced in those sessions correspond to the block 1 of the topic. In this sense, elements regarding the purposes of science teaching and learning and the concepts of scientific-technological literacy and natural environment were introduced. In practicum 1 (P1) under the name 'Laboratory as a learning space', the students, organized in work groups, had to complete the form given by the teacher. The goal of this exercise is to recognize different spaces of the laboratory, propose safety standards through the creation of a Decalogue, recognize labels of substance indicators present in the laboratory and know instruments and devices to operate with measurements and calculations of temperature, volume and mass. In practicum 2 (P2) under the name 'Do we know what we eat?' students had to design and carry out a short research about the presence of starch in food (bread, potato, apple and three different brands of ham). P2 was divided in two parts: First, practical work was more illustrative and groups had to know and apply the technique of lugol; in a second moment, groups had to strictly design a small investigation to determine the presence or absence of starch in certain foods. Finally, the different groups had to share their results, discuss with the entire class and expose conclusions. This allowed P2 to be related to the science content of the teaching and learning purposes covered by the topic in the block 1, within the context of a collaborative large group session (Figures 1A and 1B).

A) B)

Figure 1. A) Practicum 1. Laboratory as a learning space: basic safety standards and laboratory equipment; B) Practicum 2. Do we know what we eat? Research design based on the topic starch content in foods

Practicum 3 (P3) was divided into two different sessions. First, ideas about the concept of life and dead matter were explored by using a questionnaire. Complementing this, it was programmed a field work in the area that surrounds the Faculty - the natural park of 'Los Toruños'. A circuit of 30 minute route was established. During this time, the teams had to collect elements that they considered part of the natural environment. Back in the laboratory, students had to identify the elements and classify them as living and dead matter. In the second session, the work teams had to build a dichotomous key using A3 format. To do this, the teacher made a brief explanation about the use of dichotomous keys to identify and classify organisms or objects. A dichotomous key adapted to the stage of children, by López and De la Cruz (2016), was shown as an example. The key is organized into dichotomies (sometimes trichotomies) or dilemmas, i.e., pairs of opposing statements. Students, by teams, had to design a dichotomous key to classify the elements collected in the previous session. Each team presented its dichotomous key to the rest of the class, and a discussion was carried out on the elements that exist in the natural environment, bearing in mind the dichotomy of living and dead matter (Figures 2A and 2B). When designing the dichotomous key, the students had certain difficulties in assigning characteristics that allowed them to classify the collected elements. This was the case for elements such as hardness, colour or textures for inert matter. They also presented doubts between the inert concepts and “death.”.

A) B)

Figure 2. Practicum 3. What is alive? What is the natural environment made up of? A) Fieldwork in the natural park ‘Los Toruños’; B) design of a dichotomous key using collected samples from the natural environment

Data collection instrument

Once each practical work was finished (P1, P2 and P3), students, individually, had to submit a field diary and answer a specific survey for each practical work. The surveys were designed following the work by Dávila et al. (2015). Questions in every survey were adapted based on the objectives settled for each practical work. The surveys shared questions aiming at understanding the opinion of the students in every aspect. Each form contained three questions to properly address the case of study. The questions are closed, and they use a Likert scale that offers the student the possibility to choose between four different levels of agreement or disagreement.

Data analysis procedures

The data collected were processed by quantitative analysis counting absolute frequency that was performed with SPSS 21 software (mac version). A comparative analysis was carried out based on the percentages of frequencies for the three aspects considered for each practical work, designed and implemented for teaching and learning the topic DMN.

Findings and discussion

The results obtained from the analysis of the survey answers are shown in Table 1. Regarding to usefulness, results indicate that students show agreement, perceiving all the three activities as positive. However, according to the obtained data, some practical work received better scores than others. Following this, in one hand, in P1, 61% (N=51) of the students considered that they were totally in agreement with the usefulness of the activity for professional training purposes; instead P3 and P2 received 43% and 59%, respectively. P1 was the activity most valued by the students, due to their training as teachers and the so-called 'novelty factor' (Aguilera 2018; Orion & Hofstein 1994). This term refers to field trips, but can be extended to practical laboratory work, since it refers to three aspects: 1. Cognitive (concepts and skills that students should handle during the activity); 2. Geographical (the place where the activity will take place, or the workspace, the laboratory in our case); 3. Psychological (gap between expectations and the reality that students find during practical work). In this sense, the fact of using these resources so scarcely during their academic training makes the novelty factor increase. This fact could be related to a higher assessment and degree of satisfaction by students, not considering other methodological aspects.

Table 1. Survey results. Perceptions from students after the practical work applied in DMN topic during the academic year 2018-2019 (%)

Opinion	**Degree of satisfaction**	**Practicals**		
		P1 (N=51)	**P2 (N=51)**	**P3 (N=56)**
(1) Useful for professional training	Totally agree	61	43	59
	Agree	38	55	41
	Disagree	0	0	0
	Totally disagree	0	0	0
	Does not answer	1	2	0
(2) Degree of satisfaction with contents	Totally agree	29	55	38
	Agree	65	41	62
	Disagree	4	4	0
	Totally disagree	0	0	0
	Does not answer	2	0	0
(3) Utility for your future teaching work	Totally agree	53	53	36
	Agree	45	47	53
	Disagree	0	0	11
	Totally disagree	0	0	0
	Does not answer	2	0	0

The degree of satisfaction with the contents also obtained positive reactions, since the majority of the students stated that they agree or totally agree with each practical work and in no case were negative responses obtained. Although an important difference can be noticed between the experiences from P1, with a 65% of the acceptance, against P2, with 41%, and P3, with 62%. There are still aspects to highlight from each practice. P2, got a higher percentage of students who are totally satisfied with its contents than P1 and P3 did. P2 was a practice that involved instruments manipulation and specific staining techniques to assess the presence of starch in food. This practice related to food which is a subject of interest and close to students. The other two practices *a priori* do not have such a close relationship with their daily lives. Consequently, this could be the cause for the higher percentage of students who stated to be totally satisfied with P2. Finally, regarding the third aspect analysed in this research - the utility of the practice for your future teaching work - students rated P1 and P2 more positively than P3. In fact, students showed a lower level of agreement with P3 (only 11% agreed) probably due to the difficulty they felt when they were asked to design a dichotomy key.

According to the results obtained, students seem to value the usefulness of the practices for their future, as soon as they become classroom professionals, consistently with the degree of difficulty that they experienced when performing them. These results are consistent with those that emerged from other studies carried out with future primary school teachers when assessing the use of analogical resources and their transference into the primary school (Aragón, Jiménez-Tenorio, & Oliva, 2014; Jiménez-Tenorio, Aragón, & Oliva, 2016). As a conclusion, it can be said that the use of particular resources is complex or difficult for some future teachers. They rated lower the activities that they felt more difficult, as they think that when they have to transfer them to the classroom, their future students will feel the same difficulty. Students did not consider the possibility of adapting the resources to be used by children of 3 to 6 years old, neither of adapting them to the needs of prospective teachers. Besides, they did not attend to the didactic potential that this resource may offer when one wants to teach a specific science content.

Conclusion and implications for science education

In general, the results show a very positive reaction of prospective teachers to practical work as they believe that it will work as good mediator or the learning process. However, further investigation needs to be undertaken into student's perceptions of practical work, using other instruments of qualitative analysis. A good example in this regard is the use of laboratory notebooks or individual interviews, which can help to develop more insights about student teachers' perceptions. In general, practical work designed for this study seems to have awoken students' interest in science whilst developing other essential skills that are relevant for scientific research. This result has important implications in the short term, given that prospective teachers should be able to transfer these practices to their future students. Preparatory school is a fundamental educational stage of children education. In this period, one of the main objectives to be attained has to do with initiating students into scientific practices. These practices involve the development of scientific skills such as observation, variables manipulation or hypothesis formulation. In order to do this, it is necessary teachers able, not only transmit scientific knowledge, but also to promote the development of research skills. To succeed in doing so, teachers need to be able to propose understandable and searchable questions to children of early ages as a mean to lead them to investigate their natural environment and to develop an understanding of the milieu that surrounds them (Caballero & Dashoush, 2017).

An interesting aspect to be pursued in future research would be to monitor prospective teachers to find out how they are able to adapt their practices in schools during their teaching practice. Besides, it should be assessed how the end of undergraduate programme projects impact on their own interest of science.

Carrying out these studies would important because they involve actions that could lead to an increase in the number of prospective teachers who intend to teach science at the preparatory level through practical work. In addition, it could offer relevant information on how to train prospective teachers, in case the conclusion of the research is that participants demonstrate a real interest in science.

From the initial teacher education perspective, it is necessary to continue the task of planning and designing teaching proposals. These should integrate practical work into classroom strategies to promote scientific research. This should be done by carrying out small, but real, research to solve meaningful questions and problems from the participants' point of view. Practices of this type also integrate modelling and argumentation, and would allow to work with the three basic components of scientific competence.

The above arguments assume that students are able to transfer appropriate and emerging strategies and approaches, studied within the didactics of science, into professional contexts.

Finally, although the results obtained in this study have been positive, they must be considered with prudence, because assessing certain methodologies positively does not necessarily imply changes in the students' didactic models. However, the practical works do seem to promote more positive attitudes towards science, as it has been shown in this study and had been concluded in previous research (Dávila et al., 2015). Changes in prospective teachers' conceptions of how to teach and learn science are slow. Moreover, as argued by Greca, Meneses, and Diez (2017), these changes require traditional visions of science teaching and deficiencies in scientific content to be overcome. However, future teachers still keep them.

References

Abell, S. (2007). Research on science teacher knowledge. In S. Abell, & N. Lederman (Eds.), *Handbook of research on science education* (pp. 1105-1149). Mahwah, NJ: Lawrence Erlbaum.

Aguilera, D. (2018). La salida de campo como recurso didáctico para enseñar ciencias. Una revisión sistemática. *Revista Eureka sobre Enseñanza y Divulgación de las Ciencias, 15*(3), 3103.

Aragón, L., Jiménez-Tenorio, N., & Oliva, J. (2014). Evaluando una experiencia de formación inicial con maestros sobre el sistema Sol-Tierra desde la óptica de la modelización. *Actas de los XXVI Encuentros de Didáctica de las Ciencias Experimentales*, 193-200.

Caballero, A., & Dashoush, N. (2017). Planting Deeper. Outdoor experiences challenge children´s misconceptions about the needs of plants. *Science and Children*, 56-61.

Camaño, A. (2003). Los trabajos prácticos en ciencias. In M. Jiménez (Coord.). *Enseñar ciencias*, 95-118. Barcelona: Graó.

Cañal, P. (2007). Enseñar y aprender investigando. *Alambique Didáctica de las Ciencias Experimentales*, *52*, 9-19.

Cantó, J., & Serrano, N. (2017, September). ¿Cuáles son los principales problemas para hacer presentes las ciencias en las aulas de infantil?: La visión de los maestros en ejercicio. *X Congreso Internacional sobre Investigación en Didáctica de las Ciencias*, 1995-2000. Retrieved from https://ddd.uab.cat/pub/edlc/edlc_a2017nEXTRA/29_-_Formacion_en_ciencias _del_profesorado_d e_Educacion_infantil.pdf

Cantó, J., de Pro, A., & Solbes, J. (2016). ¿Qué ciencias se enseñan y cómo se hace en las aulas de ducación infantil? La visión de los maestros en formación inicial. *Enseñanza de las Ciencias*, *34*(3), 25-50.

Dávila, M., Borrachero, A., Cañada, F., Martínez, G., & Sánchez, J. (2015). Evolución de las emociones que experimentan los estudiantes del grado de maestro en educación primaria, en didáctica de la materia y la energía. *Revista Eureka sobre Enseñanza y Divulgación de las Ciencias*, *12*(3), 550-564.

Del Carmen, L. (2000). Los trabajos prácticos. In F. Perales, & P. Cañal (Coord.). *Didáctica de las ciencias experimentales: Teoría y práctica de la enseñanza de las ciencias*, 267-288. Spain: Marfil.

García Barros, S., Martínez Losada, C., & Mondelo Alonso, M. (1998). Hacia la innovación de las actividades prácticas desde la formación del profesorado. *Enseñanza de las Ciencias*, *16*(2), 353-366.

Greca, I., Meneses, J., & Diez, M. (2017). La formación en ciencias de los estudiantes del grado en maestro de Educación Primaria. *Revista Electrónica de Enseñanza de las Ciencias*, *16*(2), 231-256.

Grilli-Silva, J. (2018). El material natural en la Biología escolar. Consideraciones éticas y didácticas sobre las actividades prácticas de laboratorio. *Revista Eureka sobre Enseñanza y Divulgación de las Ciencias, 15*(1), 1104.

Hodson, D. (1992). In search of a meaningful relationship: An exploration of some issues relating to integration in science and science education. *International Journal of Science Education*, *14*(5), 541-566.

Jiménez-Tenorio, N., Aragón, L., & Oliva, J. (2016). Percepciones de estudiantes para maestros de Educación Primaria sobre los modelos analógicos como recurso didáctico. *Enseñanza de las Ciencias*, *34*(3), 91-112.

López, M., & De la Cruz, O. (2016). Colecciones y claves dicotómicas. Clasificar e identificar elementos naturales desde niños. *Alambique Didáctica de las Ciencias Experimentales*, *84*, 55-60.

Mazas, B., & Bravo, B. (2018). Actitudes hacia la ciencia del profesorado en formación de Educación Infantil y Educación Primaria. *Profesorado, Revista de Curriculum y Formación del Profesorado*, *22*(2), 285-301.

Mellado, V., Borrachero, A., Brígido, M., Melo, L., Dávila, M., Cañada, F., Conde, M., Costillo, E., Cubero, J., Esteban, R., Martínez, G., Ruiz, C., Sánchez, J., Garritz, A., Mellado, L., Vázquez, B., Jiménez, R., & Bermejo, M. (2014). Las emociones en la enseñanza de las ciencias. *Enseñanza de las Ciencias*, *32*(3), 11-36.

Murphy, C., Neil, P., & Beggs, J. (2007). Primary science teacher confidence revisited: Ten years on. *Educational Research*, *49*(4), 415-430.

Ocaña, M., Quijano, R., & Toribio, M. (2013, September). Aprender ciencia para enseñar ciencia. *IX Congreso Internacional sobre Investigación en Didáctica de las Ciencias*, 2545-2551. Retrieved from https://core.ac.uk/download/pdf/38988624.pdf

Orion, N., & Hofstein, A. (1994). Factors that influence learning during a scientific field trip in a natural environment. *Journal of Research in Science Teaching, 31*(10), 1097-1119.

Santmartí, N. (2001). Enseñar a enseñar Ciencias en Secundaria: Un reto muy complicado. *Revista Interuniversitaria de Formación del Profesorado*, *40*, 31-48. Retrieved from https://dialnet.unirioja.es/servlet/articulo?codigo=118090

Séré, M. (2002). La enseñanza en el laboratorio. ¿Qué podemos aprender en términos de conocimiento práctico y de actitudes hacia la ciencia? *Enseñanza de las Ciencias*, *20*(3), 357-368.

Deterministic conceptions about behaviour do not reflect teachers' perception of their teaching practices

[1]Mayra Antonelli-Ponti, [1]Patrícia Ferreira Monticelli & [2]Teresa Vilaça
[1]Universidade de São Paulo, Brasil & [2]Universidade do Minho, Portugal
antonelli.may@usp.br | monticel@usp.br | tvilaca@i.e.uminho.pt

Abstract: It is unclear whether deterministic conceptions have any impact on teachers' practices, and if it depends on the teacher's formation. Brazil scored low in science and mathematics in the Programme for International Student Assessment (PISA) and scored a little less in reading. We investigated the conceptions about behaviour mutability in teachers from these knowledge areas (science/mathematics teachers, SCM) and teachers of professionals of the education process (pedagogues, PED). We evaluated their conceptions about the etiology of human behaviour and about their teaching practices, and how teachers' conceptions and teaching practice affect it other. The PED tended more to a genetic perspective than SCM. Both PED and SCM disagreed with the statements geared to genetic determinism and reported a very often use of alternative strategies during their classes. We found no relation between teachers' conceptions and teachers perceptions about behaviour etiology. Teachers in our Brazilian sample are teaching with no deterministic tendency.

Keywords: basic education; beliefs; genetic determinism; nurture; secondary education

Introduction

The quality of the relationship between teachers and students and teachers conceptions affect the learning process in many ways (Castéra & Clément, 2014; Dutton Tillery, Varjas, Meyers, & Collins, 2010; Freire, 2005; Osti & Martinelli, 2014; Spinath & Spinath, 2005). Teachers conceptions, i.e., the manner how they see the life phenomena and their values, knowledge, and practices (Castéra & Clément, 2014) are reflected in their teaching practices (Buehl & Beck, 2014). The impact of teacher assessment on children self-perception is also relevant and its importance increases over the school years, at the same time the importance of parents' perception decreases (Spinath & Spinath, 2005). For instance, satisfactory performance students report being the most praised and chosen as helpers by the teacher (Osti & Martinelli, 2014), while underperforming students claim to be more criticized and declared guilty of unruly behaviour that occurs in the classroom.

In Brazil, there is a professional category in education, the Pedagogue, who is the professional responsible for the literacy on the early years of elementary education. This specialization is relevant for inclusive education, as teachers that previously consider learning disabilities and unruly behaviours as indissoluble are not able to succeed in enhancing the social and cognitive abilities of the students (Antonelli-Ponti, Versuti, & Silva, 2018; Luckesi 2011). The professional formation course of Pedagogy can change pre-conceptions with its theoretical and practical structure and knowledge (Marinho & Omote, 2017).

Behavioural genetics has brought valuable information and contributions to the educational process (Asbury & Plomin, 2013; Crosswaite & Asbury, 2018), but is not part of the pedagogues under graduation formation. Learning about genetic mechanisms may break the stigma of genetic determinism (Gericke et al., 2017), i.e., the reductionist view that genetic influence over a trait means genetic fatalism or immutability (Bernard, Dercon, Orkin, & Taffesse, 2014; Cástera & Clément, 2014), that will lead teachers to low investment in their students. In the same manner, who conceive human behaviour as highly malleable have conceptions related to environmental determinism. In education, environmental determinism may lead teachers to make excessive demands from students.

Abilities and behaviours develop by a myriad of many possibilities of interactions between organisms and its environment (Gericke et al., 2017; Briley et al., 2018). Unless teachers have this in mind and reflect about their practices and conceptions (Kreijns, Vermeulen, Evers, & Meijs, 2019), they may exert authority and power through the students that will, in turn, assume passive and submissive roles. When teachers believe and strive to extract the potential of their students, they can increase the quality of the interaction between them and develop self-confidence and high-performance students (Freire, 2005).

A proportion of teachers from different school subjects and nationalities used deterministic conceptions to explain student behaviours, whether if emerging from subjective (implicit) questions regarding genetic and environmental influences on human behaviours (Castéra & Clément, 2014), or objective questions about the same issue (Antonelli-Ponti et al., 2018; Antonelli-Ponti & Crosswaite, 2019; Walker & Plomin, 2005). Brazilian elementary school teachers pointed hereditary components as the cause of learning disabilities in Oliveira and collaborators research (2012). Their subject sample believed that the family participation in the educational process and the presence of a multidisciplinary scholar team were necessary to improve performance. The use of diversified learning strategies was not even considered as important in this process. In a different sample of Science and Mathematics Brazilian teachers, the quality of classroom interactions and their teaching strategies was pointed as a relevant influence over the learning process (Reisdoefer, Teixeira, & Ramos, 2017). Puig and Aleixandre (2015) argues that deterministic conceptions could affect even biology teachers, which should be informed about nature-nurture interaction on behaviour.

Being a human science teacher increases the weight of the scale to the side of the conception that behaviours are influenced only by genes, while being a biological or an exact science teacher weights the notion that behaviour is influenced by integrated nature-nurture mechanisms (Antonelli-Ponti et al., 2018). When considering learning disabilities alone, humanities teachers think that students are more influenced by genes than by the environment, while teachers of biological and exact sciences consider the influence of the environment rather than the influence of genes. Considering cognitive skills alone, humanities teachers attributed equal weight to genetic and environmental influences, while life sciences teachers were not associated with any conception, and exact science teachers tended to attribute these skills more to the environment (Antonelli-Ponti & Crosswaite, 2019).

Among three competencies, reading, mathematics, and science, Brazil is better at reading and shows insufficient competencies, in general, in mathematics and science (OCDE, 2016). Based on the academic performance of Brazilian students, reported by the Programme for International Student Assessment (PISA), it is challenging to understand the reasons for the worst performance of Brazilian students against others, as European countries, in these disciplines (OCDE, 2016).

Considering that in the learning process teachers involvement with the students and teachers conceptions about inclusion affect the teaching practices and the relationship with students, we include here pedagogues (PED), and science/mathematic teachers (SCM). We focus on understanding how these teachers conceive the etiology of human behaviour, how they perceive their teaching practices and the relationship between teachers' conceptions about human behaviour and their teaching practice.

Method

We conducted a cross-sectional quantitative study, with a convenience sample of teachers, after the acceptance of the Human Research Ethics Committee of the Faculty of Philosophy, Sciences and Letters of Ribeirão Preto - University of São Paulo, Brazil. The data was collected through an online questionnaire disseminated to the target audience (Science, Mathematics and pedagogues teachers) by email and on social media such as WhatsApp, Facebook, and LinkedIn, using the Google Forms platform.

Sample

The convenience sample consisted of 114 Brazilian teachers divided into two subgroups: 57 Science and/or Mathematics teachers (SCM) with an average of 38 years old (SD = 10) and 57 pedagogue teachers (PED) with an average age of 40 years old (SD = 10).

Research tools

We adopted three instruments in this study using a Likert scale of 5 points, as described below, translated by the authors.

(1) *Explicit Conceptions* (EC) About Etiology of Relevant Behaviours on Educational Environment Scale (personality, intelligence, learning difficulties, and mental disorders). This scale has five answer options, from (1) 'only genes' to (5) 'only environment' on this structure, repeated for all items/behaviours:

Please tell us how you think genes and the environment influence personality.
( ) All genes
( ) More genes than the environment
( ) About half genes, half environment
( ) More the environment than genes
( ) All the environment

Initially, this scale was used by Walker and Plomin (2005) and was adapted to Brazilian Portuguese by Antonelli-Ponti et al. (2018). On these both papers the instrument can be checked.

(2) *Implicit Conceptions* (IC) About Genetic and Environmental Influence on Human Behaviour Scale. The items are statements that consider that genes determine human traits. This scale has five options, from (1) 'totally agree' to (5) 'totally disagree', on this structure and three statements examples:

Please indicate to what extent you agree with the following statements
I. If Einstein clones could be obtained, they all would be very intelligent.
II. It is for biological reasons that women cannot hold positions of high responsibility as men can.
III. Ethnic groups are genetically different, and that is why some are superior to others.

The version applied in this study was previously used for comparisons between countries, including Brazil (Carvalho & Clemént, 2007; Castéra & Clément, 2014; Jourdan, Pironom, Berger, & Carvalho, 2013; Munoz, Bogner, Clément, & Carvalho, 2009). The instrument, including 14 items about genetic determinism, can be checked on Castéra and Clément (2014).

(3)*Teacher Perception of Teaching Practice* (PTP) Scale. The items are first-person statements about the frequency of teaching strategies used by teachers (Antonelli-Ponti, Tokumaru, Monticelli, Vilaça, & Costa, 2019). This scale has five options, from (1) 'rarely' to (5) 'often', on this structure and three statements examples:

Think about the practices you have been using in your classes during this school year. Which option represents how often you use each of the following practices/strategies?
I. I diversify the format of the classes to suit students' particularities.
II. I address questions to the students encouraging them to participate in lessons.
III. I explain the subject until all students have understood.

This questionnaire was designed by the first author as part of her doctoral project. It was inspired by the PISA questionnaire to be answered by students to know how they perceive feedback, support and adaptation of the teachers' teaching (INEP, 2016).

For both the Explicit Conceptions (EC) and the Implicit Conceptions (IC) on Etiology of Behaviours, an average scale score was calculated. A value close to 1 (one) represents genetic determinism, and close to 5 (five) represents environmental determinism. In the PTP scale, an average score close to 1 (one) indicates a low frequency of teaching strategies and close to 5 (five) indicates a high frequency of their use.

Data collection

The convenience sample was invited to participate through the dissemination of the research on social networks. The teachers who agreed to collaborate on this survey responded to an online questionnaire constructed on the Google Forms questionnaires. For this paper, we had answers from 57 science and math teachers and answers from teachers from various areas. For comparison between the knowledge areas, we chose the first 57 answers from the list of pedagogue teachers among the latter group.

Data analysis

For each research instrument, we estimated the participant's average of EC, IC, and PTP. Afterwards, we conducted a descriptive analysis with the values of the mean and standard deviation of each instrument, per group. For comparison between groups, we used the Student's t-test. The impact of teachers' conceptions on teaching practices was analysed through Pearson's' correlations coefficient and multiple linear regression analysis. Statistical analyses were performed using the statistical software IBM SPSS Inc. (International Business Machines Statistical Package for the Social Sciences, Chicago, IL, USA). For the multiple linear regression analysis, the explicit and implicit conceptions (EC and IC) were considered as independent variables, and the teachers' perception of teaching practices (PTP) as a dependent variable.

Results

The results for the EC scale shown that SCM equilibrated between genes and environmental effects over behaviour, while PED tended to focus on a genetic determinism (Table 1). This difference is statistically significant ($t\ (56) = -\ 3.854$; $p< .001$). In the IC scale, both groups disagree with the statements geared to genetic determinism, and in the PTP scale, both groups reported frequently using strategies during their classes (Table 1). There was no statistically significant difference between groups on IC and PTP scales ($p>.005$).

Table 1. Descriptive statistics of research tolls, on Science/mathematics and Pedagogues

	Science/mathematics		Pedagogues	
	Mean	SD	Mean	SD
Explicit Conceptions	3.01	.55	2.75	.50
Implicit Conceptions	3.74	.52	3.73	.47
Perception Teaching Practices	4.07	.55	4.12	.42

The product-moment Pearson's test pointed no significant correlation within the groups. There were significant negative correlations between PED implicit conceptions and the SCM explicit conception ($r = -.34$), and between PED implicit conceptions and the SCM teaching practice ($r = -.31$) (Table 2).

Table 2. Matrix of correlation of research tolls on SCM and PED

		Pedagogues			Science/Mathematics		
		EC	IC	PTP	EC	IC	PTP
Pedagogues	EC	1					
	IC	.09	1				
	PTP	.00	-.07	1			
Science/mathematics	EC	-.13	-.34*	.01	1		
	IC	-.25	-.17	-.14	.11	1	
	PTP	.03	-.31*	.20	.00	.23	1

*Correlation is significant

There were no significant correlations within the groups. Still, we decided to run the regression analysis, using the conceptions (EC and the IC) scales as independent variables, and teaching practice (PTP) scale as a dependent variable. As we expect, there was no significant relationship among science and mathematics groups, neither on the pedagogue group of teachers.

Discussion

This study analysed two ways of teachers' conceptions about genetic and environmental influences on human behaviours - Explicit (EC) and Implicit (IC) Conceptions - and teachers perception of own teaching strategies (PTP), in a sample of Brazilian teachers of Science and Mathematics (SCM) and Pedagogy (PED). On average, teachers considered both genetic and environmental influences on human behaviours, as noted before (Walker & Plomin, 2005; Crosswaite & Asbury, 2018; Antonelli-Ponti et al., 2018; Oliveira et al., 2012). PED explicit conceptions were mostly geared towards a genetic influence on human behaviour, and the opposite was seen in SCM. It is worth mentioning that, on average, PED and SCM teaching conceptions about the genetic and environmental influences on behaviour were in accordance with other scientific discoveries in this area (Briley et al., 2018; Polderman et al., 2015). However, we do not know how much those teachers know about behavioural genetics. In the UK, for example, teachers' conceptions were also balanced in the middle between the genetic and environmental axis, even when the genetic knowledge of behaviour was low (Crosswaite & Asbury, 2018).

In both PED and SCM teachers, the average values increased, from the Explicit Conceptions (EC) through the Implicit Conceptions (IC), to the Teaching Practices (PTP). The characteristics of each research instrument may explain the increase. Reporting the etiology of behaviours in a logical way (i.e., using EC and evaluating teachers balancing between the genetic and environmental weights), may intimidate less the respondents than answering questions of morality (part of the IC scale). In the case of teaching practices (PTP), each teacher perceived own practice in a particular way and supposedly assigned a high rate of application of teaching strategies to their daily school activities, inflating the results. In addition, teachers reported believing that their attitudes impact student behaviour (Dutton Tillery et al., 2010), so it is natural that they report their high commitment during teaching.

The Explicit Conceptions (EC) about the etiology of human behaviour were the only ones that differed among the groups of teachers. Science and Mathematics' teachers seem to conceive genetic and environmental influences in a balanced way for behaviours considered relevant in the educational process, and the pedagogue teachers seem to believe that such behaviours are a little more influenced by the genes. This result corroborates the findings of Antonelli-Ponti et al. (2018), and Antonelli-Ponti and Crosswaite (2019). Using the same EC questionnaire in a more robust sample (N=501), they noticed that teachers who graduated in humanities tended to weight more the genetic axis to a set of behaviours in the educational environment (Antonelli-Ponti et al., 2018; Antonelli-Ponti & Crosswaite, 2019). Perhaps the response of the biological sciences teachers, who receive information about genetics on their teacher training and then teach this subject (Puig & Aleixandre, 2015), balanced the influence of genes and nurture in SCM. Besides, pedagogues are often teaching students in childhood that is a stage at which non-cognitive skills, such as personality, are influenced more by genes than by the environment (Briley & Tucker-drob, 2014). Implicit Conceptions about the etiology of behaviour did not vary significantly between SCM and PED, as in the research involving a sample of teachers from 23 countries of different disciplines (Castéra & Clément, 2014). Both groups' conceptions varied from neutral to partially disagreement with deterministic statements about the nature of human characteristics.

Even in the face of the balanced conceptions found in these groups, teachers conceptions are not necessarily derived from scientifically accepted information and explanatory models capable of promoting self-reflection (Kreijns et al., 2019), and the understanding of the complexity of the theme (Gericke et al., 2017). The effect of teacher training courses proved to be effective in changing teachers' conceptions (Marinho & Omote, 2017), and some studies have shown that using videos as an intervention tool can be useful in reversing fatalistic behaviours (Bernard et al., 2014). A complete teacher-training course on the genetics of human behaviour focused on educational issues is already available on YouTube (bit.ly/genetcomp). A research project involvingh teachers who attended this course, in Brazil, is in progress (Antonelli-Ponti, Valenti, Díaz, Picoli, & Versuti, 2018).

Perceptions of teaching practices also do not differ between areas, which indicates that regardless of teachers' specializing area, they tend to use similar and high frequency teaching strategies. Besides, although the literature indicates possible associations between how the origin of human behaviour is conceived and everyday attitudes, such as the teacher's attitudes towards the student, no relationship among EC, CI, and PTP was found. This lack of association is not uncommon, as Buehl and Beck (2014) argue:

> "This lack of congruence is no reason to discount the power of beliefs. Instead, it is necessary to understand the potential relationship between beliefs and practice as well as the possible internal and external factors that may support or hinder this connection" (p.66).

These authors also found that no association between beliefs and practices can be explained by situations in which teachers use the teaching practices to hold the professional requirements and institutional requirements, and not just based on their views and beliefs (Buehl & Beck, 2014). This may be a reasonable explanation for the present result. However, the change in conceptions about the human behaviour over time, as argued by Antonelli-Ponti et al. (2018), through understanding that teacher attitudes influence student learning (Reisdoefer et al., 2017) and coupled with the knowledge of integral education, already advocated by Freire (2005), may be altering the dynamics of attitudes of preference by teachers to their students (Osti & Martinelli, 2014) or prejudiced attitudes and genetic determinism, at least in the educational environment (Puig & Aleixandre, 2015).

Conclusions and implications

Science and mathematics teachers, as well as pedagogues, consider the influence of genes and the environment on appropriate behaviours in the educational environment. Pedagogues attach more importance to genes than the environment to such behaviours than science and mathematics teachers, and this difference may be due to the experience of groups with students of different ages. Both groups of teachers do not agree with statements with genetic determinism about human behaviour.

Understanding the relationship between teaching conceptions and practices broadens the current knowledge about this relationship and can collaborate to make decisions in teacher training courses. Conceptions about the etiology of human behaviours are not related to teachers' perception of their teaching practices. Therefore, for this sample, we can set aside the hypothesis that these conceptions may be a problem for the educational process (even that they influence the PISA results in Brazil) and consider that these groups teach without determinism, which is a very encouraging result for the social appreciation of teachers.

Acknowledgements

This work was supported by the Higher Education Personnel Improvement Coordination - Brazil (CAPES) - under the Grant number 001, and the National Funds through FCT - Foundation for Science and Technology under the project of the Research Centre on Child Studies of the University of Minho (CIEC) with the reference UID/CED/00317/2019.
We thank the teachers who participated in this study.

References

Antonelli-Ponti, M., & Crosswaite, M. (2019). Teachers' perceptions about the etiology of intelligence and learning difficulties. *International Journal of Educational Psychology*, *8*(4), 162-187.

Antonelli-Ponti, M., Tokumaru, R., Monticelli, P., Vilaça, T., & Costa, P. (2019). *Development and Validity Evidence of the Teaching Practice Perception Scale*. Manuscript submitted for publication.

Antonelli-Ponti, M., Versuti, F., & Silva, J. (2018). Teachers' perception about genes and behavior. *Estudos de Psicologia Campinas*, *35*(4), 421-431.

Antonelli-Ponti, M., Valenti, L., Díaz, D., de Picoli, R., & Versuti, F. (2018). Divulgação científica em psicobiologia: Educação à distância como estratégia para a promoção da extensão universitária. *Expressa Extensão*, *23*(3), 183-197.

Asbury, K., & Plomin, R. (2013). *G is for genes: The impact of genetics on education and* *achievement* (Vol. 24). Chichester: John Wiley & Sons.

Bernard, T., Dercon, S., Orkin, K., & Taffesse, A. (2014). *The future in mind: Aspirations and forward-looking behaviour in rural Ethiopia.* London: Centre for Economic Policy Research.

Briley, D., Livengood, J., Derringer, J., Tucker-Drob, E., Fraley, R., & Roberts, B. (2018). Interpreting behavior genetic models: Seven developmental processes to understand. *Behavior Genetics*, *49*(2), 196-210.

Briley, D., & Tucker-Drob, E. (2014). Genetic and environmental continuity in personality development: A meta-analysis. *Psychological Bulletin*, *140*(5), 1303.

Buehl, M., & Beck, J. (2014). The relationship between teachers' beliefs and teachers practices. In H. Fives, & G. Gill (Eds.) *International handbook of research on teachers' beliefs* (pp. 66-84). Abingdon: Routledge.

Carvalho, G., & Clément, P. (2007). Projecto 'Educação em Biologia, Educação para a Saúde e Educação Ambiental para uma melhor cidadania': Análise de manuais escolares e concepções de professores de 19 países (europeus, africanos e do próximo oriente). *Revista Brasileira de Pesquisa em Educação em Ciências*, *7*(2), 1-21. Retrieved from https://periodicos.ufmg.br/index.php/rbpec/article/view/4036

Castéra, J., & Clément, P. (2014). Teachers' conceptions about the genetic determinism of human behaviour: A survey in 23 countries. *Science & Education*, *23*(2), 417-443.

Crosswaite, M., & Asbury, K. (2018). Teacher beliefs about the aetiology of individual differences in cognitive ability, and the relevance of behavioural genetics to education. *British Journal of Educational Psychology*, *89*(1), 95-110.

Dutton Tillery, A., Varjas, K., Meyers, J., & Collins, A. (2010). General education teachers' perceptions of behavior management and intervention strategies. *Journal of Positive Behavior Interventions*, *12*(2), 86-102.

Freire, P. (2005). *Pedagogia do oprimido*. Rio de janeiro: Paz e Terra.

Gericke, N., Carver, R., Castéra, J., Evangelista, N., Marre, C., & El-Hani, C. (2017). Exploring relationships among belief in genetic determinism, genetics knowledge, and social factors. *Science & Education*, *26*(10), 1223-1259.

INEP, M. (2016). Brasil no PISA 2015: Análises e reflexões sobre o desempenho dos estudantes brasileiros/OCDE-Organização para a Cooperação e Desenvolvimento Econômico. São Paulo: Fundação Santillana.

Jourdan, D., Pironom, J., Berger, D., & Carvalho, G. (2013). Factors influencing teachers' views of health and health education: A study in 15 countries. *Health Education Journal*, *72*(6), 660-672.

Kreijns, K., Vermeulen, M., Evers, A., & Meijs, C. (2019). The development of an instrument to measure teachers' inquiry habit of mind. *European Journal of Teacher Education*, *42*(3), 280-296.

Luckesi, C. (2011). *Avaliação da aprendizagem: Componente do ato pedagógico*. Cortez: São Paulo.

Marinho, C., & Omote, S. (2017). Concepções de futuros professores a respeito da Educação Inclusiva e Educação Especial. *Revista Educação Especial*, *30*(59), 629-642.

Munoz, F., Bogner, F., Clément, P., & Carvalho, G. (2009). Teachers' conceptions of nature and environment in 16 countries. *Journal of Environmental Psychology*, *29*(4), 407-413.

OCDE. PISA 2015: Technical Report (2016). Retrieved from https://www.oecd.org/pisa/data/2015-technical-report/. Access in April 2017.

Oliveira, J., Santos, S., Aspilicueta, P., & Cruz, G. (2012). Teachers’ conceptions about learning difficulties. *Revista Brasileira de Educação Especial*, *18*(1), 93-112.

Osti, A., & Martinelli, S. (2014). Desempenho escolar: Análise comparativa em função do sexo e percepção dos estudantes. *Educação e Pesquisa*, *40*(1), 49-59.

Polderman, T., Benyamin, B., de Leeuw, C., Sullivan, P., van Bochoven, A., Visscher, P., & Posthuma, D. (2015). Meta-analysis of the heritability of human traits based on fifty years of twin studies. *Nature Genetics*, *47*(7), 702-709.

Puig, B., & Aleixandre, M. (2015). El modelo de expresión de los genes y el determinismo en los libros de texto de ciencias. *Revista Eureka sobre Enseñanza y Divulgación de las Ciencias*, *12*(1), 55-65. Retrieved from https://revistas.uca.es/index.php/eureka/article/view/2902

Reisdoefer, D., Teixeira, E., & Ramos, M. (2017). A influência da relação professor-estudante na aprendizagem discente: Percepções de professores de ciências e matemática. *Revista Exitus*, *7*(3), 64-87.

Spinath, B., & Spinath, F. (2005). Development of self-perceived ability in elementary school: The role of parents’ perceptions, teacher evaluations, and intelligence. *Cognitive Development*, *20*(2), 190-204.

Walker, S., & Plomin, R. (2005). The Nature-Nurture Question: Teachers’ perceptions of how genes and the environment influence educationally relevant behaviour. *Educational Psychology*, *25*(5), 509-516.

L. Leite et al. (Eds.) (2019), Proceedings of the ATEE Winter Conference 'Science and mathematics education in the 21st century'
Brussels: ATEE and CIEd (ISBN: 9789082506549)

Teacher-students learning to notice children's mathematical reasoning in a video club

Loek Spitz &Monique Pijls

Centre for Applied Research in Education (CARE), Amsterdam University of Applied Sciences, The Netherlands

L.Spitz@hva.nl | m.h.j.pijls@hva.nl

Abstract: One of the core principals of primary school mathematics in the Netherlands is that teachers should guide the learning process of their pupils. To be able to do that, teachers need to notice their pupils' mathematical thinking; that way they can adapt their guidance to each individual pupils need. However, the skill to notice mathematical thinking is usually not emphasized in Dutch teacher training programs for primary school. In this paper, we study video clubs as a means to train Dutch pre-service teachers to notice their pupils' mathematical thinking. The results indicate that video clubs can indeed be used to help pre-service teacher to notice mathematical thinking, and also to realize the importance of this noticing. As an unintended consequence of the video club meetings, the participants also discovered the usefulness of asking their pupils how they arrived at their answers.

Keywords: mathematical thinking; primary school teacher training; realistic mathematics education; video club

Introduction

In the Netherlands, just like in many other Western countries, there is discussion about the best way to teach mathematics, especially for children in primary school (Gravemeijer, Bruin-Muurling, Kraemer & van Stiphout, 2016). The method for teaching primary school mathematics that has been prevalent for the last thirty to forty years, called Realistic Mathematics Education (RME), has come under pressure. At least in part, this is due to the Netherlands scoring lower than before in international comparison studies of mathematical ability in primary school, like PISA (OECD, 2016) and TIMSS (Mullis, Martin, Foy & Hooper, 2016). For some years now, a discussion has been ongoing between supporters of RME and supporters of a more 'traditional' pedagogy of mathematics, based on instruction and repetition (van de Craats, 2007; Uittenbogaard, 2007), an in-depth analysis of which is beyond the scope of this introduction. The Royal Dutch Academy of Sciences conducted a thorough investigation into the matter in 2008 and came to the conclusion that while there was no strong scientific evidence for either form of pedagogy of mathematics producing better results, the quality of the teacher does have a direct impact on the mathematical ability of the pupil (KNAW, 2009). Because RME is currently the prevalent pedagogy of mathematics in the Netherlands, it makes sense to investigate ways to improve teacher quality within the framework of this pedagogy.

One of the core tenants of RME is the guidance principle. Van den Heuvel-Panhuizen (2000) explains this as "students should be given a guided opportunity to reinvent mathematics. The teacher steers the learning process, but not in a fixed way by demonstrating what the students have to learn.". However, to be able to guide children in their reinvention of mathematics, it is necessary to identify and understand their problem-solving strategies. This requires teachers to notice and analyse their pupils' mathematical thinking.

According to Levin, Hammer and Coffey (2009) noticing pupil mathematical thinking is not only important to be able to adapt to it, but also because discussing mathematical thinking can put the focus of a lesson on mathematical reasoning itself, instead of learning some specific mathematical skill. In other words, they "care about teacher attention not only for the information it provides teachers but also for how it models a key aspect of scientific reasoning for students" (p. 143). Such constant noticing and analysing of their pupils' ideas can be quite demanding and is usually not expected of novice teachers. Levin et al. (2009) argue, however, that it is possible for novice teachers to learn and apply these skills, and therefore, they should be taught in teacher education.

In the experience of the authors, noticing of pupil mathematical thinking, especially during normal classroom teaching, is usually not emphasized in Dutch primary school teacher training programs. Therefore, it is worthwhile to research ways in which learning to notice and analyse mathematical thinking could be implemented in Dutch primary school teacher training. In the current paper, we look into one such way, which is to use a video club. This idea is based on a series of papers by Sherin and van Es (2008, 2009, 2010), wherein they study the possibilities of using a video club to help mathematics teachers to notice mathematical thinking. We repeat their research in a different setting; where they studied in-service high school mathematics teachers in the US, we look at pre-service primary school teachers in the Netherlands. An important difference between these contexts is that primary school teachers do not specialize in mathematics; they are general teachers that have to be able to teach any primary school subject, and often have no special interest or aptitude for mathematics.

Theoretical framework

Mathematical learning is a human activity for which reflective communication is fundamental (Freudenthal, 1991; Mason, Burton, Stacey, 2010; van Oers, 2010). Hence, pupils should be encouraged to vocalize their thinking while working on mathematics. However, the skill to notice and analyse a pupil's mathematical thinking is usually not emphasized in Dutch primary school teacher training. It is important to find ways to help teacher-students to analyse their pupils' mathematical reasoning, and to learn to recognize the importance of such analysis. For children, learning arithmetic and mathematics is a complex process that requires development of thinking (Cai & Knuth, 2011). Thinking out loud and discussions may help them to structure their ideas.

For the teacher, it is important to stimulate thinking out loud and to adapt instruction to their pupils' development (Walshaw & Anthony, 2008). This starts with *noticing* children's mathematical thinking, which is a professional action that can be learned and taught. Professional noticing of children's mathematical thinking consists of three activities (Jacobs, Lamb & Philips, 2018). To notice children's mathematical thinking teacher must first of all pay attention to children's strategies. The recognition of the ideas and thoughts of children is the first step in helping them to structure their complex thoughts. Secondly, they have to interpret the ideas of the children to construct a 'picture' of children's thinking based on snapshots of their strategies. Lastly, they have to decide how to respond, given this interpretation.

When teachers learn to notice children's mathematical thinking a shift is visible in their behaviour from general remarks towards specific remarks on students' mathematical understanding and from generalizations in understanding towards details on a specific situation of students (Jacobs, Lamb & Philips, 2018).

These shifts in teachers' behaviour require reflection on their professional action. A way to achieve this kind of reflection is in a video club setting, where teachers jointly discuss videotaped lessons with colleagues (Borko, Jacobs, Eiteljorg, Pittman, 2006; Gaudin & Shelies, 2015). The effects of video clubs on teachers' professional development can be positive, especially when they watch their own lessons and when the discussions in the video club are led by an expert (Beisiegel, Mitchell & Hill, 2018; Roth et al., 2017). Sherin and van Es (2009) used the concept of video clubs to help teachers in US schools to notice and analyse their students' mathematical thinking. At each of the ten meetings fragments of videotaped lesson taught by the participants were viewed and discussed. The discussions were led by one of the researchers, who used question to direct the participants' attention to pupil mathematical thinking. After one year of professionalization participants reported that they better attended to their pupils' mathematical thinking in the classroom and were better able to interpret their reasoning. This was reflected by a shift in the video club discussions, where they were indeed more attentive of pupil mathematical thinking, and in their teaching as observed in the classroom, where they made more room for their pupils to express their thinking and used it more.

We are interested to find out whether video clubs can be used in a similar way with Dutch students in teacher training, which leads to the following research question: Can video clubs be used to teach students who are learning to become primary school teachers to analyse their pupils' mathematical thinking and to learn to recognize the importance of such analysis?.

Note that this question pertains to two 'levels' of learners; that is, children in primary school learning mathematics and students in university learning to become teacher. To avoid confusion, we use 'pupil' when referring to the first and 'student' when referring to the second.

Research method

This paper describes a study monitoring a video club organized at the primary school teacher program of the University of Amsterdam. The goal of the video club was to teach participating students to notice and analyse their pupils' mathematical thinking. The data obtained for the study consisted of entrance and exit interviews and observations of the video club meetings.

Participants

All students (around 100 in total) in the second, third and fourth year of the primary school teacher education program of the University of Amsterdam were invited to participate in the video club. Initially five students signed up, but one cancelled after the first meeting due to lack of time. Two of the participants where in the second year of the four-year program, two in the fourth. All students in this program take a one day per week internship throughout the program, students in the fourth year take a two days per week internship.

The participants made the videos to be discussed in the video club at their internship. Students in this program often videotape their lessons for discussion during the program, but these discussions are usually aimed at classroom management and general pedagogy, not pupil (mathematical) thinking.

The interviews

Participants were interviewed individually before and after the video club series. In both interviews, the participant watched three clips from a primary school mathematics lesson. Each clip lasted around five minutes and contained some pupil mathematical thinking. Participants were allowed to take notes while watching the clips. After each clip, the participant was asked "What did you notice?" after which they were given as much time as they needed to describe everything that was of note to them. When they seemed to be done, they were asked "Anything else?" and given the opportunity to add something more. When the participant was done, the next video was started and the same process was repeated. The total time between the entrance and exit interviews was a little over three months.

The video club meetings

The video club met four times in a period of three months. Each meeting lasted about two hours with a short break in the middle. Video material from two students was discussed at each meeting, thus allowing two filmed lessons to be discussed for each student. Two students had their video material discussed in the first and third meeting, the other two students had their material discussed in the second and fourth meeting.

The students videotaped their own lessons specifically for viewing during the video club meetings. They filmed the whole lesson (usually between 20 and 40 minutes) using one or two cameras. The fragments to be discussed where chosen by the facilitator and by the student who made the video. The fragments chosen by the facilitator were picked because they showed some pupil mathematical thinking. The students could choose any fragment they wanted to discuss with their fellow participants.

The discussions about the video fragments where of an open nature. The facilitator asked the participants to watch the video fragments and then comment on anything they noticed in the video. The students would be allowed to discuss anything they found interesting about the video. The facilitator participated as a natural participant in these discussions.

When the students had no more remarks about the video fragment, the facilitator would draw their attention to any pupil mathematical thinking that was not yet discussed. Usually this was done by asking questions like "What if we look at this pupil? What can you say about him?" or "From the things that this pupil says, what can we say about his level of doing additions? What do you think he is able to do, and what does he still have to learn?" Sometimes the facilitator would ask the students to watch part of a fragment again, paying special attention to one of the pupils where he knew some interesting mathematical thinking would be visible.

Data analysis

All interviews and video club meetings were videotaped. These videos were transcribed and divided into "idea units", as described in Jacobs and Marita (2002) and done in Sherin and van Es (2008). This was done by the first author. All the idea units were coded using five categories, adapted from Sherin and van Es (2009). The categories were: general pedagogy and classroom management (general), mathematical pedagogy (pedagogy), description of pupil mathematical thinking (describe), analysis of pupil mathematical thinking (analyse) and desire and means to make pupil mathematical thinking visible (make visible).

The general category was used for any statements regarding general pedagogy, classroom management or the general structure of the lesson. For example: "the pupils exhibited a calm demeanour while working on this problem" or "I am asking a lot of questions". The pedagogy category was used for statements specific to mathematical pedagogy, such as "I decided to use the number line here". The categories describe and analyse where used for discussions of pupil mathematical thinking. Describe was used when the students simply pointed out the mathematical thinking of a pupil, as in "she is saying it is four plus three", "they are just guessing now", or "I am impressed that he immediately sees that the answer is four". Analyse was used when the students analysed the mathematical thinking of a pupil, i.e. tried to reason how a pupil was thinking beyond simply stating what was visible: "I think the reason this pupil is saying 35% instead of 15% is that he is looking at the wrong scale; he does not understand that he should use the other one." Finally, the make visible category was used when students discussed methods to make pupil mathematical thinking visible to the teacher, for example: "He doesn't ask her what she is thinking. I think he could have asked that; that way he would have known why she answered '85%'". This category was added because it turned out to be a major take-away for the students; they explicitly named it so during one of the meetings as well as individually in the exit-interviews. Therefore, it seemed important to include in the quantitative analysis.

All interviews and video club meetings were coded by the first author. The second author coded two of the video club meetings and two of the interviews. These were compared and found to have 0.64 inter-coder reliability. One explanation for the low score is that it is almost impossible to cut the text into units where only one idea is discussed; often, two or more ideas are entangled in such a way that it impossible to separate them. In such a case the coders have to decide what idea is the most important or prevalent one in the idea unit, which can be quite subtle. An example is the following discussion in the third meeting, during which the participants were trying to figure out how a certain pupil was using his fingers to represent numbers:

Holly:	"Maybe what is going on, maybe his [the pupils] idea, I think the book is 'Wereld in getallen' ['World in Numbers', a Dutch math book for primary school], right?
Donna:	"Yes."
Holly:	"I think at some point they use pictures of fingers [to represent numbers], and I think they might use a similar hand-position. I know because I also taught that lesson, and I was wondering then why they used fingers in that way."

On the one hand, this part of the discussion seems to be about the specific method that is used to teach number representation using fingers, which is mathematical pedagogy. On the other, Holly is using this idea to try to explain why one of the pupils is doing this representation in a certain way, which is analysis of pupil mathematical thinking. Any such disagreements were discussed and resolved through consensus.

Statistical methods were used to determine whether the participants learned to notice and analyse their pupils' mathematical thinking. In particular, a *z*-test was used to determine if there was a significant increase in the proportion of idea-units that was used to discuss pupil mathematical thinking between the entrance and exit interviews. The same was done for earlier and later video club meetings. Because we expect this proportion would increase, a one-tailed *z*-test for independent samples was used. Thus *z*-values greater than 1.645 indicate a significant difference at the 0.05 level.

Results and conclusions

The results showed that with each new meeting the attention to pupil mathematical thinking increased, while the amount of general comments decreased. On average, students also showed more focus on pupil mathematical thinking in their final interview.

Analysis of the video club meetings

As shown in Table 1, in the first meeting of the video club, around 65% of the discussion was about general aspects of the lesson, 8% about mathematics pedagogy and 26% was about pupil mathematical thinking. In the later meetings, general comments decreased to around 15 - 20%, whereas mathematics pedagogy (around 30%) and student mathematical thinking (around 40 - 55%) increased.

Table 1. Video club participants' focus during the meetings

	First meeting % (f)	**Second meeting % (f)**	**Third meeting % (f)**	**Fourth meeting % (f)**
General	65 (94)	36 (57)	21 (29)	13 (14)
Pedagogy	8 (12)	21 (34)	33 (47)	29 (32)
Describe	18 (26)	27 (43)	33 (46)	40 (44)
Analyse	6 (9)	11 (17)	13 (18)	15 (16)
Make visible	2 (3)	6 (9)	1 (1)	3 (3)
Total	100 (144)	100 (160)	100 (141)	100 (109)

Note: The numbers in parentheses indicate the number of idea units coded according to a particular category. Corresponding percentages are also noted.

During the final meeting, one of the participants was absent due to illness, which might explain the lower total amount of remarks during this meeting. For this reason, we looked at the difference between the first and third meeting to determine whether the students' ability to notice and analyse their pupils' mathematical thinking increased during the meetings. We used a *z*-test to compare the number of remarks coded as describing and analysing at the first and third meetings. Both turned out to have significantly increased at the 0.05 level; for describe the one-tailed *z*-test statistic was 2.8, for analyse it was 1.9.

In the first meeting, many of the comments made by the students were general in nature, like: "They are working so quietly." Other comments were focused on the teacher: "Looking back at it, I think I am doing really well, asking a lot of questions, I'm not just explaining how they should do the exercises. I'm asking the kids; how should this be solved?" or on general pedagogy: "I am wondering about these two girls; they are finished. Now they have to wait, which is sort of a waste of time. I never know what to do about that." There are comments about pupil mathematical thinking, but they are often very general, like "Most of the kids are not able to do this exercise here". Such comments do require a participant to look at the mathematical thinking of the pupils, but without analysing why they are not able to do it, and without looking at individual pupils; they are commenting about the pupils as a group.

A few discussions during the first meeting did go deeper into analysis of pupil mathematical thinking. For instance: "It takes him a while to get the answer to 5 - 3, but once he does, he knows the answer to 15 - 3 instantly. That means he understands the connection between the two sums, but he probably still computes 5 - 3 by counting." This is a good example of a discussion where careful observation of one specific pupil led to a better understanding of his ability at that point. It must be noted that this discussion only happened after some prompts by the facilitator to look closely at what the pupil was doing.

In the third and fourth meeting, analysis happened more often and needed less prompting from the facilitator. Like this remark about a pupil using an abacus that one of the students made immediately after watching one of the fragments: “It is impressive to me that these pupils are able to see that, okay, here are two cookies and there three cookies, and now I see these five beads on my abacus and thus there must by five cookies in total. They are able to make the connection between the beads and the cookies.” Later, they get into a more involved discussion about a different pupil’s use of the abacus:

Holly: “I think this girl, in the pink, on the left, I think she gets it. Because she moved the right beads in the right direction.”
Donna: “Yes, she moves them all at once in the right direction”.
Holly: “And she writes down the answer immediately”
Donna: “Yeah, but notice that she first moves one a little bit, then a second, and then she moves them all the way over together. So she takes them one, then two-”
Holly: “She puts them sort of in the middle, so that she can check that there are actually two beads before shifting them all the way. Which is totally fine.”

Analysing the behaviour of the pupil together, Holly and Donna gain a better understanding of the level at which the pupil can operate an abacus.

One interesting thing that came up during the first meeting was a discussion about how to figure out the mathematical thinking of a pupil when it was not yet visible. The group was discussing whether a pupil had immediately seen the answer to 6 + 2, or had used the time between seeing the sum and being asked to answer it to count to the answer in her head. The students were unable to tell form the video, so the facilitator asked them what the teacher could have done during the lesson to find out. One of the participants had the idea to “give them a different, but similar sum”. The facilitator had a different suggestion: “You could also just ask; ‘how did you see that?’ That is really fast, and if they say ‘I just saw’, then you can probably believe that. Then you know, and you also gave the pupil a good feeling.” The reason this small discussion was so interesting for the study is that the participants seemed to take it to heart. In one of the lessons that was filmed for the second meeting, Holly, who was teaching that lesson, asked her pupils multiple times “How did you know?”. During the meeting she explains: “I thought, last time we discussed, how can you know how a child arrived at an answer? And now I was going to film, so I thought, I should just ask them.” This idea sometimes came back during the discussions in the meeting, but it mainly influenced the lessons the students were teaching, and thus the actual filmed material that was discussed.

During the exit interviews the students really came back to this; they now criticize the teacher in the fragments discussed there for not asking his pupils how they arrived at an answer often enough.

Analysis of the interviews

As shown in Table 2, the participants’ focus during the entrance interviews was mostly on general and pedagogical aspects of the lessons, while less than a quarter was focused on pupil mathematical thinking. During the exit interviews, more than 30% of comments was directed at pupil mathematical thinking, with another 16% focused on making this thinking visible.

Just as with the meetings, we see a decrease in the proportion of comments of a general nature and an increase in comments analysing the mathematical thinking of a pupil. This increase in analysis turns out to be significant at the 0.05 level; the *z*-test statistic relating to the difference in analysis between the entrance and exit interviews is 2.5. Contrary to what happened at the meetings, we see no increase in the proportion of comments made describing pupil mathematical thinking.

Table 2. Video club participants' focus during the interviews

	Entrance interview % (f)	**Exit interview % (f)**
General	42 (45)	22 (26)
Pedagogy	36 (38)	31 (36)
Describe	18 (19)	16 (19)
Analyse	5 (5)	15 (17)
Make visible	0 (0)	16 (18)
Total	100 (107)	100 (116)

Note: The numbers in parentheses indicate the number of idea units coded according to a particular category. Corresponding percentages are also noted.

During the entrance interview, the students did not talk about how to make the mathematical thinking of the pupils visible. At the exit interview, 18% of their comments was aimed at this. Mainly, they criticized the teacher for not asking his pupils how they had arrived at their answers, such as Regina: "He does not ask what she is thinking. And maybe that would have been useful, because he does ask the boy that said 75% [the correct answer], and he does discuss what the boy did. But I think he should have done that with the girl as well, to figure out how she arrived at 85%". Mary has a similar idea: "I think what he could do better is, he could be asking each time 'how did you see that?', or 'why do you think that is the answer?'". Holly says: "He corrects the children every time they get something wrong, he does it in such a way that they are able to give the correct answer. He does not ask why they gave that answer." Donna stresses the importance of asking even when the pupil gave a correct answer: "And only when someone says '75%' [which is wrong] he asks 'why 75?', even though he could have done that with all the answers. He doesn't ask how they got their answers, while for instance, one kid at some point answers '10%', but before he did you already heard other kids whispering 'ten, ten', so he has no idea whether the kid actually understood or not."

Conclusion

In the meetings and interviews, there is a significant increase in discussion of pupil mathematical thinking over time. Furthermore, the discussions seem to be of a deeper, more analytical level in the later meetings and exit interviews. On top of that, the participants seem to have realized the importance of making the mathematical thinking of their pupils visible by asking them questions. This follows from their own comments during the meetings, a change in the teaching that they submitted for review and from an increase in comments on this topic between the entrance and exit interviews.

This gives a good indication that it should be possible to use a video club setting to teach students learning to become primary school teachers to notice their pupils' mathematical thinking, to analyse it, and to recognize the importance of doing so. Furthermore, a video club also seems to be a way to teach them a practical tool to make such thinking visible in asking their pupils short follow up questions of the form 'how did you arrive at that answer?'.

Discussion

This paper reports on an exploratory study, meant to figure out whether it is worthwhile to experiment further with video clubs as a means to teach future primary school teachers to analyse their pupils' mathematical thinking. It expands upon research of Sherin and van Es (2008, 2009, 20010) by translating their ideas to the Dutch system, to pre-service teachers, and to primary school teachers, which are not subject specialists but generalists. It is important to expand their research in this way, because the prevalent way of teaching mathematics in Dutch primary schools requires teachers to know and understand their pupils' mathematical thinking in such a way that they are able to give each pupil the individual guidance they need. From the research described in this paper, we conclude that video clubs indeed seem to be a way in which future primary school teachers can be taught this skill.

At this point, it is important to note that the lessons that were discussed in the video club meetings might have changed over time as the participants learned to care more about pupil mathematical thinking. In that case, the reason the participants noticed more pupil mathematical thinking might not have been a result of them learning to notice, but instead a result of them teaching in a way that makes pupil mathematical thinking more visible. In fact, it seems that this is at least partially what happened, as evidenced by the discussion described on page 7, where Holly states: "I thought, last time we discussed, how can you know how a child arrived at an answer? And now I was going to film, so I thought, I should just ask them." However, this cannot fully explain the results of this paper because there was also a significant increase in noticing of mathematical thinking between the entrance and exit interview, which used identical film fragments. Furthermore, it seems to us a very promising result for teacher educators that a video club might not only be used to help future primary school teacher to notice their pupils mathematical thinking, but also to change their lessons in such a way that this mathematical thinking becomes more obvious.

The following things should also be considered when assessing the results of this study. The first is the low inter-coder reliability. Even though this is partly explained by the subtleties of coding complicated discussions, it does detract from the results somewhat. In our view, in this case it is good enough, because this study is more exploratory in nature. For a larger scale purely quantitative study, this would need to be addressed.

Secondly, in this study the first author was also the facilitator in the video club meetings and one of the two people to code the transcriptions. This might have had some influence on the meetings themselves, and also meant that it was not possible to code without knowledge of which meeting was being coded. Again, we feel that for an exploratory study, where a large part of the impact should be in the descriptions of the video club meetings and in inciting further, large scale quantitative studies, this is good enough.

Lastly, only four students participated in the video club, and they were not randomly selected. All students in the second, third and fourth year of the primary school teacher education program of the University of Amsterdam were invited to participate. Certain kinds of students are more likely to participate, for instance those with an interest in mathematics or those more involved in the study program. Therefore, the results of this study cannot be generalized to all students in such a teacher education program.

Implications

To gain a better understanding of the way video clubs can be used to help teachers notice and analyse their pupils' mathematical thinking there are a couple of different directions, that should be explored. Scientifically, it makes sense to repeat a similar study on a larger scale, with randomized controlled trials. Additionally, the long-term effects of such video club meetings should be researched, as well as the actual effect on the participants' teaching. Finally, one should note that this research was carried out with university students. Only a small part of students in teacher training in the Netherlands receive their education at a university; most are in vocational schools. Research should be conducted to find out whether the results described here generalize to those students.

For application in teacher training, it would be useful to explore the role of the facilitator. Recall that in the current study, the facilitator choses most of the video fragments discussed, and also directs the participant's attention to any pupil mathematical thinking the participants might have missed. It is not feasible to use this model if one were to implement video clubs into teacher training for all students. Therefore, it makes sense to see whether some parts of the facilitator role could be taken up by some of the participants themselves without losing the positive effects of the video club. We feel that at least part of having an expert in the pedagogy of mathematics as a facilitator is important, as the students seemed not to focus on their pupils' mathematical thinking at all in the first meeting, unless prompted by the facilitator. Still, it might be possible to gain similar effects even with a less involved facilitator, for instance by modelling the facilitator role in a larger setting before students have their own video club meetings in smaller groups. This is the approach taken by Klabbers (2016), which is worth exploring further.

Another way to avoid high costs in facilitating video clubs for all students would be to make them available as electives only. As it appeared in this study, that would limit the number of participants significantly, thereby allowing for a more involved facilitator. However, that would limit the effects to a small group of students, and would still be relatively expensive.

Based on the results of this study, we do recommend teachers in pre-service (or in-service) primary school teacher training who are struggling to get their students to notice and use the mathematical reasoning of their pupils to try out using video clubs in the way described in this paper.

References

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education*, *24*, 417-436.

Beisiegel, M., Mitchell, R., & Hill, H. (2018). The design of video-based professional development: An exploratory experiment intented to identify effective features. *Journal of Teacher Education, 69*(1), 69-89.

Cai, J., & Knuth, E. (2011). *Early algebraization. A dialogue for multiple perspective.* New York: Springer.

Van de Craats, J. (2017). Waarom Daan en Sanne niet kunnen rekenen. *Nieuw Archief van de Wiskunde, 5/8*(2), 132-136.

Freudenthal, H. (1991). *Revisiting mathematics education.* (China lectures). Dordrecht: Kluwer.

Gaudin, C., & Chalies, S. (2015). Video viewing in teacher education and professional development: A literature review. *Educational Research Review, 16*, 41-67.

Gravemeijer, K., Bruin-Muurling, G., Kraemer, J., & van Stiphout, I. (2016). Shortcomings of mathematics education reform in the netherlands: A paradigm case?, *Mathematical Thinking and Learning, 18*(1), 25-44.

Jacobs, V., Lamb, L., & Philipp, R. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169-202.

Jacobs, J., & Morita, E. (2002). Japanese and American teachers' evaluations of videotaped mathematics lessons. *Journal for Research in Mathematics Education*, *33*(3), 154-175.

Klabbers, V. (2016). Beeldbegeleiding als opleidingsdidactiek. *Velon Tijdschrift Voor Lerarenopleiders*, *37*(3), 73-78.

KNAW (2019). *Rekenonderwijs op de basisschool, analyse en sleutels tot verbetering*. Amsterdam: Koninklijke Nederlandse Akademie van Wetenschappen.

Levin, D., Hammer, D., & Coffey, J. (2009). Novice teachers' attention to student thinking. *Journal of Teacher Education*, *60*(2), 142-154.

Mason, J., Burton, L., & Stacey, K. (2010). *Thinking mathematically*. Harlow: Prentice Hall.

Mullis, I., Martin, M., Foy, P., & Hooper, M. (2016). *TIMSS 2015 International Results in Mathematics*. Retrieved from http://timssandpirls.bc.edu/timss2015/international-results/

OECD (2016). *PISA 2015 Results (Volume I): Excellence and Equity in Education*, PISA, OECD Publishing, Paris.

Roth, K., Bintz, J., Wickler, N., Hvidsten, C., Taylor, J., Beardsley, P., Caine, A., & Wilson, C. (2017). Design principles for effective video-based professional development. *International Journal of STEM Education*, *4*(31), 1-24.

Sherin, M., & van Es, E. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education*, *24*(2), 244-276.

Sherin, M., & van Es, E. (2009). Effects of video club participation on teachers' professional vision. *Journal of Teacher Education*, *60*(1), 20-37.

Uittenbogaard, W. (2007). Hoe Juliette en Jonas leren rekenen - Appels en Peren. *Panama Post*, *26*(1), 32-36.

Van Es, E. & Sherin, M. (2010). The influence of video clubs on teachers' thinking and practice. *Journal of Mathematics Teacher Education, 13*, 155 - 176.

Van den Heuvel-Panhuizen, M. (2000). *Mathematics education in the Netherlands: A guided tour. Freudenthal Institute Cd-rom for ICME9.* Utrecht: Utrecht University.

Van Oers, B. (2010). Emergent mathematical thinking in the context of play. *Educational Studies in Mathematics, 74*, 23-37.

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research, 78*(3), 516-551.

L. Leite et al. (Eds.) (2019), Proceedings of the ATEE Winter Conference 'Science and mathematics education in the 21st century'
Brussels: ATEE and CIEd (ISBN: 9789082506549)

Student teachers´ needs and preferences for a technology preparation course

Sonia Palha
Amsterdam University of Applied Sciences, The Netherlands
s.abrantes.garcez.palha@hva.nl

Abstract: In this study, we investigate, through a questionnaire, in-service student teachers' needs and preferences in a master program for a course about learning with technology. Results show that about half of the student teachers would like to have such a course and about one third has doubts about it. The reasons provided by students with different needs and their preferences for the content, learning activities and teacher support are discussed in this paper, as well as implications for the design of teacher preparation programs.

Keywords: in-service teacher education; teacher preparation program; technology integration; TPCK

Introduction

There has been much debate about what in-service student teachers need to know with regard to learning with technology and how they are supposed to develop this knowledge (Koehler & Mishra, 2005). Teacher education programs need to focus 'on developing an understanding of the complex set of interrelationships between artefacts, users, tools, and practices.'. In other words, developing technological pedagogical content knowledge (TPCK). Research (Fu, 2013; Tondeur, van Braak, Ertmer, & Ottenbreit-Leftwich, 2017) shows teacher preparation programs have beneficial influence on teachers' attitudes and beliefs about the use of technology in practice. However, this does not mean teachers change their practice. In fact, it remains a challenge to turn positive intentions and TPCK-knowledge into classroom practice.

As many other higher education institutions, the master teacher education department of the Amsterdam University of Applied Sciences (AUAS) struggles with this problem. Until 2018, the subject digital pedagogies was part of the discipline mathematical pedagogy and there was no specific course for preparing in-service student teachers for learning with technology. In addition, there were no defined directions within the institution to guide the teacher educators in the development of such a program. For that reason, the department decided to investigate student teachers' needs for a teacher preparation course about TPCK-knowledge and use the results to design a new course.

In this paper, the results of part of the larger research are presented and discussed. They provide insight in student teachers' needs and dispositions for developing TPCK-knowledge that can be of interest beyond our institution.

Theoretical background

Development of TPCK

The TPCK framework (Koehler & Mishra, 2005) emphasizes the interplay of three bodies of knowledge needed for a good integration of technology into practice: knowledge about content (C), pedagogy (P), and technology (T). Technological pedagogical content knowledge is a kind of knowledge that goes beyond the use of the three components. It e.g. requires an understanding of the representation of concepts using technologies; pedagogical strategies; knowledge about students' pre-knowledge and difficulties and how technology can help some of the problems that students face. High-qualified teachers should have an understanding of the relationships between these matters and be able to use this understanding in their practice.

In this line of thought, the development of TPCK-knowledge in teacher preparation programs should consider the TPCK-components in relation to each other and not in isolation. Moreover, learning to reason explicitly about how ICT can support specific pedagogical strategies can contribute for this to happen (Heitink, Voogt, Fisser, Verplanken, & van Braak, 2017). However, as we discuss next, there are other aspects that influence the development of TPCK-knowledge.

Factors that matters in technology integration

In a research review about how teachers integrate technology in the classroom Fu (2013) points a number of factors influencing technology integration in practice. These factors were grouped in external and internal factors. *External factors* include technology availability, technical support, time to develop instruction, curriculum, school culture, and pressure to prepare students for exams. *Internal factors* include knowledge, understanding and skills of technology use, beliefs and attitudes toward technology integration, intention or motivation to use ICT and technology self-efficacy.

According to the review, the most common external factors impeding technology integration included lack of access to computers and software, insufficient time to plan instruction and inadequate technical and administrative support (Chen, 2008). Some external factors were positively associated with technology integration such as the availability of technology and technical support. Fu claimed that technology availability and support are important to technology integration.

Among the internal factors, the most relevant factors are teachers' beliefs and confidence in utilizing technology and understanding about technology use. Chen (2008) findings show that in some cases teachers lack of theoretical understanding could explain the inconsistency between teachers expressed beliefs and the teachers' practices. For example, almost all teachers in Chen's study reported high agreement with constructivist concepts, however, in practice teachers implemented the policies on the basis of their own interpretation and understanding. Chen also found that teachers' beliefs and contextual factors may affect each other. For example, a school culture that uses tests to motivate students will reinforce some teachers' traditional beliefs that value knowledge transmission. Teachers' beliefs concern also the discipline they are teaching. In a multiple case study, Niess (2005) examined the TPCK of five pre-service student teachers who followed a science and mathematics teacher preparation program. He found out that their development of TPCK was related with how they view the integration of technology and the discipline they were teaching. These and other contextual factors may explain some teachers' unwillingness or hesitation to allow students spend more time exploring content on their own with technology. Therefore, variables such as teachers' attitudes, beliefs and confidence can be seen as strong predictors for teacher and student technology use.

Tezci (2011) investigated the role of internal factors and external factors on the level of technology usage by pre-service teachers. As Chen (2008), the researcher found out that internal and external factors were related to each other and to technology usage level. For instance, teacher's perception (internal factor) of school climate and support were found to be relatively low. Support is however considered to be an external factor that influence effective technology integration. Perceived support in Tezci's (2011) study was also found to be correlated with internal factors, including attitudes towards computers and the Internet, self- confidence, and knowledge. The researcher concludes that: "effective ICT integration requires a school culture and support (an external factor) that provides its pre-service and in-service teachers with the necessary knowledge and experience (internal factors) regarding effective and successful ways to integrate ICT into classroom activities" (p. 496).

Several researchers have investigated the link between teachers' pedagogical beliefs and their educational uses of technology. In a recent review of research on this matter, Tondeur et al. (2017) selected and analysed 14 studies. Their findings suggest that the relationship between pedagogical beliefs and technology use comprise a bi-directional relationship. Based on these studies they claim that "the integration of technology in classroom educational processes has the potential to change teachers' beliefs towards more student-centred, constructivist beliefs. But also, constructivist beliefs lead to use of technology that support the development of 21st century skills." Thus, technology can also be beneficial to teachers with teacher-centred pedagogical beliefs. Within these results, the authors remark it is important to note the iterative process of learning to teach with technologies: beliefs lead to actions and actions lead to the development of reconstructed beliefs. They conclude that, regardless of teachers' pedagogical approaches, teachers find value in using technology when it aligns with their current pedagogical approaches. Therefore, technology should be introduced in teacher preparation programs in ways that align with their actual approaches and values because this increases the likelihood that teachers will integrate and use technology in their practice.

Technology preparation or programs and teacher education

Research (Fu, 2013) also shows that internal factors and relationships between internal and external factors can be influenced through participation in technology preparation courses or programs. For instance, after a semester-long technology literacy course, the preservice teachers in Abbott and Faris (2000) developed more positive attitudes towards computers because of the instructional approaches, meaningful assignments requiring technology, and supportive faculty. See Fu (2013) for more examples.

Teacher preparation programs can also impact technology integration by engaging teachers in design-based activities (Laurillard, 2012; Koehler & Mishra, 2005) because it helps teachers to develop a flexible and situated knowledge of the value of technology for learning. In this approach, teachers may work collaboratively to develop technological solutions for authentic pedagogical problems or in re-designing their lessons or course materials in ways that the use of technology improves learning.

Research questions

Tondeur et al. (2017) suggest that technology should be introduced in ways that align with teachers' current practices. Based on our theoretical framework and following this suggestion we investigated what are the needs of in-service student teachers for a technology preparation course during the study. The research questions that guide the investigation are: what is the need for in-service student teachers of mathematics for a course about digital pedagogy during their study? (RQ1); what is in-service student teachers' preference for the content, learning activities, and type of teacher support? (RQ2).

Method

Participants

The participants were students of a three-year master course for Mathematics Teacher in secondary education. The in-service student teachers teach mathematics at the secondary level. They typically work 3-4 days a week in a school and follow the master teacher program one day a week, on Thursday. All student teachers (*N*=84) were invited to participate in the research by their teacher and/or coordinator of the department via email. About 40% of the student teachers (*n*=34) took part in the study.

Instrument and data collection

An online questionnaire was used to investigate the research questions. The questionnaire was developed by the researcher together with two other teacher educators. It consists of 16 open and multiple-choice questions, in which respondents may add information and explain their answer if they wish to. The questionnaire investigated: student teachers' use of technology in their education (questions 1-5); *needs for a course* on digital didactics (question 6); their *preference for content, learning activities, form* and *guidance* of the course (questions 7-13) and suggestions and wishes for a measure of involvement in the design process (questions 14-16). I report in this paper only the results of questions 6-13 because they concern the research questions. The other results are published in the project report, which can be requested from the author.

Data analysis

The results of multiple-choice questions are automatically translated into tables and graphs. By the analysis of the open questions and explanations by the multiple-choice questions, we applied open coding in the following way: (i) The author and a student assistant encoded all answers independently of each other and defined provisional categories; (ii) the coding and the categories were discussed and adjusted; (iii) the author re-coded all answers according to the agreed categories and, where needed, the categories were more sharply defined; (iv) the coding was discussed with the student assistant and adapted until agreement was reached.

Results

In-service student teachers need for a course digital pedagogies

The question about students' need of a course about digital pedagogies was investigated with a multiple-choice question followed by a request to explain their choice (question 6). The students could select one of the following options: "Yes, I want to follow the course"; "I doubt about it"; "No, I don't want to follow the course." Thirty-two students of the thirty-four answered the question and twenty-two explained their choice. These explanations were analysed as described in the method section. We came to a number of categories presented together with the results in Table 1.

Results show that about half (53%) of the student teachers would follow the course. Most of the reasons presented by students were related to internal factors. Like to improve knowledge or ability. An example of an answer of this type is:

'I am curious about the opportunities that I do not know."

Table 1. Question 6, students' needs and reasons to follow a course

Answers choice	**Students (*n*=32)**	**Reasons presented (categories)**
Yes, I want to follow the course	17 (53%)	more knowledge / skills (*n*=6) practical examples to use (*n*=1) hints to apply school wide (*n*=1) other (*n*=1)
I doubt	11 (34%)	depending on the offer (*n*=3) little time/ workload (*n*=3) doubt the benefits of a course (*n*=2)
No, I don't want to follow the course	4 (13%)	little time/ workload (*n*=3) attends further training or has support at school (*n*=1) already feels competent (*n*=1)

Other reasons concerned appliance to school practice:

> "You take the time to deepen your knowledge, and you will probably get a lot of material."
> "I would like to hear new ideas and suggestions on how to apply them school-wide."

About one-third (34%) of the student teachers doubts about following a course. Three students would attend the course if the content suits their interest and level. One student wrote:

> "As long as it is deepening for me. I want to know the content of the course beforehand, on the basis of which I would make a choice."

Three students mentioned little time/workload:

> "I doubt because I have little time for a course. I would rather take such a course when I have finished my study."

And two students mentioned having doubts about the benefits of a course because it will probably not solve the problems they experience. For instance, one student refers to the value of the course with regard to acquiring more knowledge, but it will not help the lack of technical affordances at his school:

> "Often the digital boards do not work or hardly do it. The question next is it worth to make two variants of a lesson in case the technology doesn't work? On the other hand, I find it interesting to know more about the possibilities of technology."

Only four students stated not intending to follow the course. Two of these students referred to little time/workload. One student that his school already provides support:

> "In the school where I teach much attention is paid to this."

One student considers himself already competent and does not feel the need to learn more about it.

Preferences for content and learning activities

The students' preference for content and learning activities was investigated through questions 7, 8 and 9. All three questions were multiple-choice with the possibility to choose one or more options and to explain the given answer.

At question 7, three options for the course content were given: practical knowledge and abilities; background and theoretical knowledge; technical abilities. Results showed that almost all students (94%) prefer practical knowledge. Technical abilities were chosen by about half of the students (52%) and theoretical knowledge was the less chosen content (39%).

Question 8 investigated in more detail student's preference for content by asking about the subject matters that could be handled in the course. Based on the literature we pulled up a list of possible topics. Figure 1 presents these topics and summarises the answers of the thirty-two students who answered the question. The two most chosen subjects were learning about tools for mathematical learning (74%) and designing lessons for learning with technology (65%). Topics regarding the use of technology for assessment (summative or formative) were chosen also by many of the respondents. The less chosen topics were open online education, chosen by one single student and innovative or adaptive technology (four students).

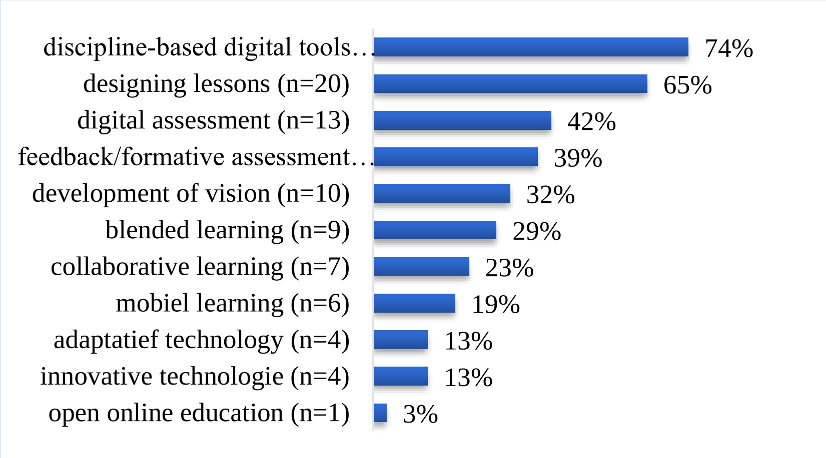


Figure 1. Question 8 (What subjects would you like to see in the course?)

Question 9 investigated the students' preference for learning activities. The students could choose more than one activity from the list presented in Figure 2. None of the 30 students who answered the question added other learning activity. The only student who added a comment stated that the activities should be related to mathematics. The most chosen activities were developing digital education (70%) and following guest lectures (53%). The less chosen were participation in webinars and reading texts (two students).

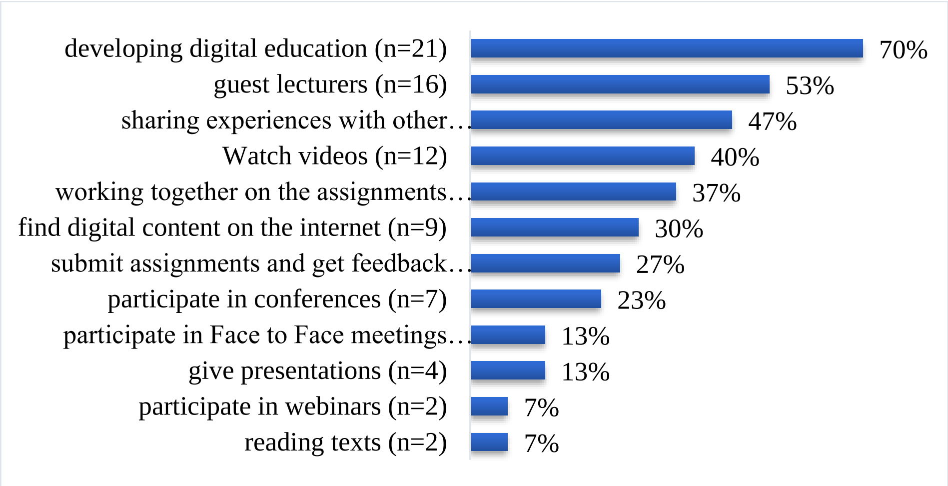


Figure 2. Question 9 (What learning activities would you like to see in the course?)

Preferences for teacher support

When asked about what kind of teacher support would you like to see in the course (Question 13, open question) students mentioned: face-to-face guidance (27%); lectures/coaching led by an expert or practice-oriented person (23%); and involving feedback from the teacher (23%). Less mentioned forms of support involved blended and online guidance (respectively 4% and 12%) and interaction with others (8%). These results suggest students have a preference for face-to-face forms of learning. Some students explicitly stated to prefer someone from mathematics: "From an expert in technologies in mathematics (not in general)". The results are presented in Figure 3.

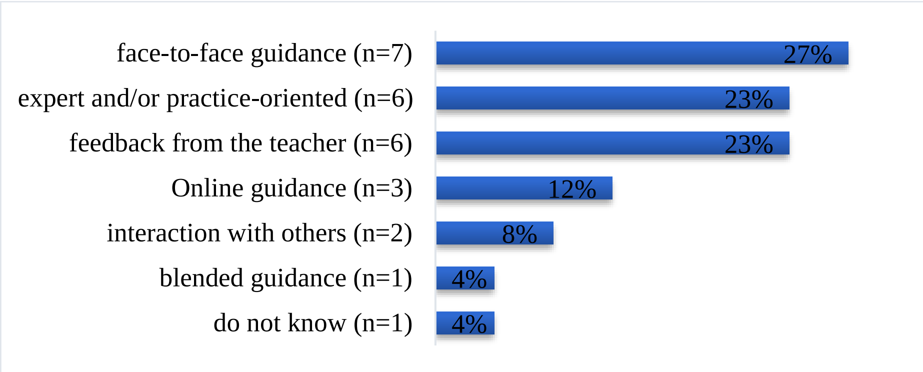


Figure 3. Question 13 (What kind of teacher support would you like to see in the course?)

Other results

Questions 10-12 inquired students' preferences for the form and duration of the course. The results are very varied and some students found it difficult to give beforehand how long the course should be.

Discussion

The role of internal and external factors in students' needs for a course

Our first research question is about the needs of in-service student teachers of mathematics for a course about digital pedagogy during their study. Understanding in-service student teachers' needs can help to choose for content and strategy. It also provides a more significant insight on students' knowledge, problems, and shortcomings. The results show that about half of the students would follow the course.

The motivation to follow, or not, a course is very varied. Looking at the results through the lens of internal and external factors, we found that all the reasons presented by students who wish to follow a course concerned internal factors. Those were: increasing or deepening knowledge or skills, getting materials and practical examples to use and hits to apply school wide. Students who doubt about following a course referred to both external and internal factors. External factors were little time, workload and already following training at the school. Internal factors referred to having doubt about the benefits of a course, depending on the content or because they already feel competent. These results are in line with previous research. Insufficient time to plan instruction is one of the most pointed external factors that influences technology integration (Fu, 2013; Chen, 2008) and motivation, need of knowledge and self-efficacy are internal factors referred in several studies (Fu, 2013). Our results suggest external and internal factors may play a different role in student teachers' motivation to follow a course. For instance, the factor time is repeatedly mentioned in a negative way and as an external factor while the factor knowledge is frequently used in a positive way and as internal factor. However, one could equally see the factor time as positively influencing technology integration by thinking about having enough time for or wanting to expend more time with learning with technologies. Fu (2013) points out that external and internal factors can be both negatively and positively associated with technology integration. Moreover, internal and external factors are related and influence each other.

Putting research into practice (part 1)

As mentioned before, it remains a challenge to turn positive intentions and TPCK-knowledge into classroom practice (Tondeur et al., 2017). Higher education can contribute to this by developing efficient teacher preparation courses that assist teachers to establish the connection between theoretical and practical knowledge. Based on our results, I suggest including thinking about the interplay between internal and external factors in teacher preparation programs. Especially during the course for in-service student teachers, the terminology could be used as a framework to think about possible factors that play a role by their own and each other's motivation. In an initial stage this could be used as a way to provoke awareness with the expectation that at long term, thinking in these terms would help teachers to develop own strategy to cope with internal and external factors and when possible turn negative factors into more positive ones. Naturally, I am aware that this will not be enough for some teachers who work in schools with a considerably diverse pedagogical orientation. As Cobb, Zhao, & Dean (2009) state: "[...] it is important to take account of the institutional setting in which the collaborating teachers work when pursuing such an agenda because the professional development goals conflict with the current instructional priorities of most schools and districts' (p. 192).

Some suggestions provided by Cobb et al. (2009) on how to work with school administrators to advance student-teacher ability to address internal and external factors include: opportunities to participate in formal professional development activities, opportunities to participate in informal professional networks and assistance from a school-based mathematics coach.

Preferences for the design of a course from a students' perspective

The second research question investigated student teachers' preferences for the content of a course, learning activities and type of teacher support. The most preferred course design included content knowledge much close to student teachers' actual practice, such as discipline-based tools, designing lessons and assessment. More innovative themes like new technologies and open online education were the less chosen subjects. The fact that these themes may be too far from student teachers' daily practice can be a reason behind their choice. Another reason can be that the student teachers due little time and high workload (typically they teach 3-4 days a week during their study) give priority to themes that they find usable for practice above more innovative themes. These results are in line with Niess (2005) results in which teachers' beliefs about the integration of technology into practice was a critical factor. Teachers who had difficulty in integrate technology into practice also had difficulty in recognizing the value of this technology for their teaching. This does not imply, of course, that the in-service student teachers are not open to innovative practices. In fact, the majority of the reasons given by the student teachers who want to follow a course were the development of new knowledge.

Putting research into practice (part 2)

Some questions on what new teachers typically need for technology integration include recognizing the interplay between the technology and content domain and teaching with a focus on students learning (Niess, 2005). As Niess points out at the start of a student teaching experience, the student teachers are usually focused on their own teaching and they think less about their students' learning and thinking. Teacher preparation programs should challenge their students to go beyond their 'habitual' way of thinking. Taking this in consideration and based on student teachers' preferences, we propose a number of directions for the design of a teacher preparation course.

In the first place, the content-knowledge and learning activities should be predominantly practice-oriented and theoretical or technical parts of the course should be explicitly connected or embedded in practice. In this way, we expect that students recognize the interplay between technology and mathematical content. Moreover, by proposing activities that emphasize their student's interpretation of the concepts when using technology, we expect that student teachers focus on students' understandings.

Secondly, it could be possible for student teachers to select technological knowledge that is relevant to them. A way to make this possible is to propose assignments that can be approached at different levels. Also, the duration of the course should meet the goals and content of the subject matter, so for learning more theoretical oriented (parts of) subjects it is suitable to have lectures of an exact duration in a certain time and for more technical or practical subjects, it can be more adequate to allow extended sessions of one full or half a day and space in time.

Finally, student teachers have a clear preference for a teacher educator experienced in the use of technology. This can be achieved through e.g. encouraging teacher educators to regularly and reflectively experiment with innovative technologies and pedagogies within mathematics education, collaborate with other teacher educators and professionals and inviting experts for lectures. Teacher educators can be challenged to experiment with innovative technologies and pedagogies for instance when enrolling in technology-based curriculum-reform projects or when collaborating with innovative schools or network of schools that plan for technology integration.

Final remarks

Concluding, in this study we discuss the results of a questionnaire about in-service student-teachers' need and preference for a course about digital pedagogies; we suggest a number of directions for developing a course based on these results and offer some recommendations to put these results into practice. One primary result is the suggestion to include thinking about the interplay between internal and external factors in teacher preparation programs. Moreover, the study provides empirical evidence that many teachers (in our case in-service student teachers) need opportunities to deep and extend their TPCK-knowledge. In-service student teachers' preference for a course seems to be practiced-based guided and innovative content is much less preferred than more traditional content and learning activities. These results are in line with previous research on teachers' integration of technology in classroom practice (Niess, 2005) and they extend these results for in-service student teachers.

This study also has some limitations. The results refer to a single teacher education department in one university and therefore should be carefully interpreted when transferred to other contexts. Another limitation concerns the closed character of a questionnaire. Although there were open questions and it was possible for the respondents to explain or extend their answers, it was not possible for the researchers to question through, as it would be possible in interviews. In spite of these limitations, we believe our findings can be useful for teacher educators and researchers interested in developing and revise their courses. They can also be useful for faculty managers who plan for curriculum development.

References

Abbott, J., & Faris, S. (2000). Integrating technology into preservice literacy instructions a survey of elementary education students' attitudes toward computers. *Journal of Research on Computing in Education*, *33*(2), 149-161.

Chen, C. (2008). Why do teachers not practice what they believe regarding technology integration?. *The Journal of Educational Research*, *102*(1), 65-75.

Cobb, P., Zhao, Q., & Dean, C. (2009). Conducting design experiments to support teachers' learning: A reflection from the field. *The Journal of the Learning Sciences*, *18*(2), 165-199.

Fu, J. (2013). Complexity of ICT in education: A critical literature review and its implications. *International Journal of education and Development using ICT*, *9*(1), 112-125.

Heitink, M., Voogt, J., Fisser, P., Verplanken, L., & van Braak, J. (2017). Eliciting teachers' technological pedagogical knowledge. *Australasian Journal of Educational Technology, 33*(3), 96-109.

Koehler, M., & Mishra, P. (2005). Teachers learning technology by design. *Journal of Computing in Teacher Education, 21*(3), 94-102.

Laurillard, D. (2012). *Teaching as a design science: Building pedagogical patterns for learning and technology*. London: Routledge, Taylor and Francis.

Niess, M. (2005). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and Teacher Education*, *21*(5), 509-523.

Tezci, E. (2011). Factors that influence pre-service teachers' ICT usage in education. *European Journal of Teacher Education*, *34*(4), 483-499.

Tondeur, J., van Braak, J., Ertmer, P., & Ottenbreit-Leftwich, A. (2017). Understanding the relationship between teachers' pedagogical beliefs and technology use in education: A systematic review of qualitative evidence. *Educational Technology Research and Development*, *65*(3), 555-575.

L. Leite et al. (Eds.) (2019), Proceedings of the ATEE Winter Conference 'Science and mathematics education in the 21st century'
Brussels: ATEE and CIEd (ISBN: 9789082506549)

Interdisciplinary tasks: Pre-service teachers' choices and approaches

Helena Rocha
UIED, Faculdade de Ciências e Tecnologia, Universidade NOVA de Lisboa, Portugal
hcr@fct.unl.pt

Abstract: This study focusses on the criteria used by pre-service teachers of Mathematics to choose interdisciplinary tasks. The pre-service teachers' knowledge is assumed as the basis of the actions taken and used as the origin of the choices and approaches observed. The study adopts a qualitative and interpretative methodology and the data were collected using class observation and interviews. The analysis is guided by the Application and Pedagogical Content Knowledge, a model inspired on TPACK (from Mishra and Koehler) and MKT (from Ball and colleagues). The conclusions point to an appreciation of the mathematical part of the tasks and to a devaluation of the remaining components. This suggests difficulty in articulating and integrating different domains of knowledge and points to a fragmented view of the potential of using mathematical applications.

Keywords: applications; mathematics; pre-service teachers; teachers' knowledge

Introduction

Most of the problems we can find in the reality are related to more than one area of disciplinary knowledge. This means that they can be considered as interdisciplinary problems. However, when these problems are approached in school, that approach tends to focus on the knowledge of one specific subject. Many times, the procedural aspects of that specific subject become the central part of the problem and everything else is ignored, simplified or even removed from what was a real situation. The result can be a task that is no longer a problem, neither an interdisciplinary task.

In this study, a set of tasks with a real context was given to a class of pre-service teachers taking a course on didactics of mathematics at a master program for future secondary mathematics teachers. The pre-service teachers were asked to choose three tasks from that set of tasks and develop a lesson plan explaining how they will use the tasks in lessons of mathematics.

The main goal of the study is to characterize the pre-service teachers' choice of tasks and the related professional knowledge. It was specifically intended to: (1) Identify the criteria used by the pre-service teachers to select the tasks; (2) Analyse what these options suggest in terms of their professional knowledge. Pre-service teachers' knowledge is assumed as the basis of the actions taken; an analysis of the tasks chosen and the related reasons are assumed as a way to access their professional knowledge.

Theoretical framework

Interdisciplinarity

The literature offers a variety of ways of understanding interdisciplinarity. Drake (1991) considers the integration of two or more disciplines, assuming interdisciplinarity as one type of integration. And Williams et al. (2016) describe this type of integration as an approach where two or more disciplinary contents are considered at the same time. This means that we are considering mathematics and some other(s) content(s) simultaneously, while addressing a specific topic or theme, but in such a way that all the disciplines keep their specific nature. Other types of integration are present, for instance, in a multidisciplinary or transdisciplinary approach (Williams et al., 2016).

Roth (2014, p. 317), states that: "interdisciplinarity denotes the fact, quality, or condition of two or more academic fields or branches of learning". So, in this case, the meaning ascribed to interdisciplinarity is similar to the one ascribed to integration by Drake (1991), referring in a broad way to a situation where some kind of integration between disciplines occurs, regardless of the characteristics of that integration. This means that depending on the author, integration can be a synonymous of interdisciplinarity or interdisciplinarity can be a special kind or level of integration.

The diversity of ways how two disciplines can be articulated was the focus of attention of several authors, who devoted their work to analyse its characteristics and develop classifications. That is the case of Fogarty (1991), who considered ten levels, according to the degree of articulation between the disciplines: fragmented, connected, nested, sequenced, shared, webbed, threaded, integrated, immersed, and networked.

Mathematical modelling or application is another term usually associated with interdisciplinarity. These two terms are often considered together; similarly, in this paper we shall not distinguish them. However, Ferri and Mousoulides (2017) alert us to the need to reflect carefully on the understanding of these two concepts - modelling/application and interdisciplinarity -, in order to identify what is common to them and what is different. The authors consider that the usual definitions of these terms show strong overlaps, not always making clear the differences. Mathematical modelling or application is often presented as an activity involving articulation between reality and mathematics (Ferri & Mousoulides, 2017). In addition, this means that modelling/application always requires a real-life context, but real-life situations tend to be related to some other discipline besides mathematics. In this sense, modelling/application is always an interdisciplinary activity. According to Ferri and Mousoulides (2017), modelling/applications are activities set within authentic contexts, and offering opportunities to engage in important mathematical processes, such as describing, analysing, constructing, and reasoning.

Nowadays, STEM has become the ultimate form of interdisciplinarity (Williams et al., 2016). However, STEM and interdisciplinarity are not the same thing. Bergsten and Frejd (2019) present STEM as an interdisciplinary approach to learning, based on real world situations from Science, Technology, Engineering, and Mathematics. This means STEM presupposes interactions within a specific set of disciplines, leaving out all the others, such as the ones related to, for instance, Humanities or Economics. Bergsten and Frejd (2019) defined STEM as based on real world situations, and Ferri and Mousoulides (2017) characterize mathematical modelling/application as an activity involving articulation between reality and mathematics. However, once again, not all the real situations are related to Science, Technology, and Engineering, what means that STEM is also not the same as modelling. Therefore, STEM is an interdisciplinary approach, often requiring some modelling process.

Here we will assume a mathematical application as a task requiring some articulation between reality and mathematics, which will include some kind of interdisciplinary approach.

Teachers' knowledge

Several authors have developed models of teachers' knowledge, identifying and characterizing different domains that integrate that knowledge (Ruthven, 2011). As a consequence, there are several characterizations of teacher professional knowledge developed over the years but, whether we base ourselves on the work of Shulman (1986) or on some other more recent work, all characterizations emphasize the importance of different types of teachers' knowledge.

Hill and Ball (2009) draw on Shulman's work and conceptualized Mathematical Knowledge for Teaching (MKT), where they consider two major areas: SMK - Subject Matter Knowledge and PCK - Pedagogical Content Knowledge (see Figure 1). In the scope of the first, they consider CCK - Common Content Knowledge, SCK - Specialized Content Knowledge and KMH - Knowledge at the Mathematical Horizon. Moreover, in the context of the second, they consider KCS - Knowledge of Content and Students, KCT - Knowledge of Content and Teaching and KC - Knowledge of Curriculum.

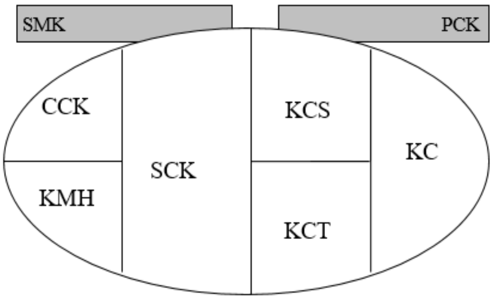


Figure 1. MKT - Mathematical Knowledge for Teaching (Hill & Ball, 2009)

In SMK, Ball, Thames, and Phelps (2008) consider Common Content Knowledge (CCK), which is identical to the knowledge used in other professions where mathematical knowledge is required. It includes knowledge of the definition of a concept or object, how to perform a procedure, how to solve mathematical tasks correctly, and even identify the correctness of an answer given by others. The authors also consider a teachers' specific knowledge, the Specialized Content Knowledge (SCK). This is a core knowledge in this model and corresponds to a knowledge that is used in the classroom and that is necessary for the teacher to be able to teach efficiently. It corresponds, for example, to the knowledge needed to identify the source of some error. A third type of knowledge was introduced later by Hill and Ball (2009), the Knowledge at the Mathematical Horizon (KMH). It corresponds to what they describe as a kind of peripheral vision needed to teach. This knowledge encompasses a broad and comprehensive overview of the mathematics teaching landscape, including an awareness of the mathematical topics covered in previous years and how they relate to those approached at the present and in the future.

In addition to these domains of knowledge, Ball, Thames, and Phelps (2008) and Hill and Ball (2009) also refer to a certain integration of content knowledge with another type of knowledge. They thus refer to three domains that we approach here in a very brief way: Knowledge of Content and Student, Knowledge of Content and Teaching, and Knowledge of Content and Curriculum.

Knowledge of Content and Students (KCS) combines knowledge of students and mathematics. This knowledge corresponds to the ability to anticipate students' difficulties, respond to their thoughts and respond to them in a timely and convenient manner. It also includes making an appropriate choice of examples and representations to teach. Both in the preparation process and in the course of its implementation, the teacher must be aware of the conceptions held by the students regarding the topic under study and, in particular, regarding the students' misconceptions.

Knowledge of Content and Teaching (KCT) articulates knowledge about mathematics and teaching. It refers, among other elements, to the teacher's decisions regarding the sequence of activities, his awareness of the possible advantages and disadvantages of the representations used for teaching, his decisions about when to interrupt a discussion in class to clarify some aspects or to use a student's opinion, and also about how to call attention to a certain mathematical aspect.

Teachers' knowledge also includes knowledge of the curriculum, as well as how the different contents interrelate and evolve throughout the school year syllabus.

One of the main points of this model is the way how it emphasizes the mutual influence among different domains of the knowledge. That is, for instance, how the teachers' knowledge of the students impacts the mathematical content and how the teaching approach also impacts the mathematical content.

The integration of something new on the teachers' practice has proved to be challenging and to require some change on the professional knowledge. One of the most studied situations is the integration of technology. In this case, the need for a different knowledge is deeply recognized and the starting point for the development of some models intending to characterize the teachers' knowledge required to integrate it. One of the most well-known models in these circumstances is the TPACK from Mishra and Koehler (2006).

Mishra and Koehler (2006) argue that the articulation of technology knowledge with other types of knowledge is fundamental. According to the authors, the relationships between content, pedagogy and technology are complex and take multiple forms. Indeed, on one hand, technology has its own imperatives that affect the content to be addressed and its representations and, on the other hand, it interferes with instructional options and other pedagogical decisions. Thus, decision making regarding the use of technology has implications for other areas and, as such, it does not seem appropriate to consider it in isolation from pedagogical knowledge and content knowledge. Mishra and Koehler (2006) then propose a model that not only considers the three referred domains of knowledge (basic knowledge), but also addresses the connections, interactions and constraints that are established between them. Thus, they consider a Technological Pedagogical and Content Knowledge (TPACK), which is based on content knowledge, pedagogical knowledge and technological knowledge, but also respond to the influences of each of the basic knowledge on each other. They thus refer to Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK) and Pedagogical Content Knowledge (PCK). These three areas of knowledge are the essence of this model and are what truly distinguishes it from others previously proposed.

Content Knowledge (CK), Pedagogical Knowledge (PK) and Pedagogical Content Knowledge (PCK) are, taking into account the origin of the model, consistent with the respective notions presented by Shulman and well documented in the literature.

Technology Knowledge (TK) involves the capabilities required to operate a technology and essentially consists of knowing how it works.

Technological Content Knowledge (TCK) is directly linked to how technology and content influence each other. It is a knowledge that, while relying on content knowledge, is different from this. Access to technology not only allows access to different representations, but also facilitates the connection and transition between them. As so, the teacher needs to know not only the content to be taught but also how it can be modified by the use of technology.

Technological Pedagogical Knowledge (TPK) is a knowledge related to the potentialities of technology and the way how teaching can be changed according to the use of it. It includes understanding how a given technology can enhance the accomplishment of a certain type of task, becoming familiar with a set of strategies that allow the students to exploit technology's capabilities, and knowing how to tailor certain teaching methods to integrate technology.

Technological Pedagogical and Content Knowledge (TPACK) is a knowledge developing from the three base components of the model (content knowledge, pedagogy and technology), but goes beyond these. This knowledge is different from that held by a mathematician or a technology expert and equally distinct from the general pedagogical knowledge shared by teachers of different subjects. It is the basis of effective technology integration and requires an understanding of concepts within technology and an understanding of pedagogical techniques that use technology constructively to teach concepts. It also requires a knowledge of what makes a concept difficult or accessible, and how technology can be used to promote students' learning. It also requires a sense of students' prior knowledge and how they learn, as well as a knowledge of how technology can be used to develop existing knowledge or to achieve new knowledge.

Quality teaching thus requires the development of an understanding of the complex relationships between the three base knowledge of the model and the ability to use that understanding to develop an appropriate and context-specific set of strategies.

At TPACK model, besides considering the relevance of the mathematical knowledge, of the pedagogical knowledge and of the technology knowledge, the mutual influence among these domains of knowledge is central. This means that when the technology becomes available, the teacher needs to consider the way it can impact the mathematics and the pedagogical approach. In these circumstances, it seems reasonable to admit that introducing mathematics applications requires some similar development of the teachers' professional knowledge. As so, we propose the model APCK - Application and Pedagogical Content Knowledge (see Figure 2), a conceptualization of the teachers' knowledge similar to the one developed at TPACK.

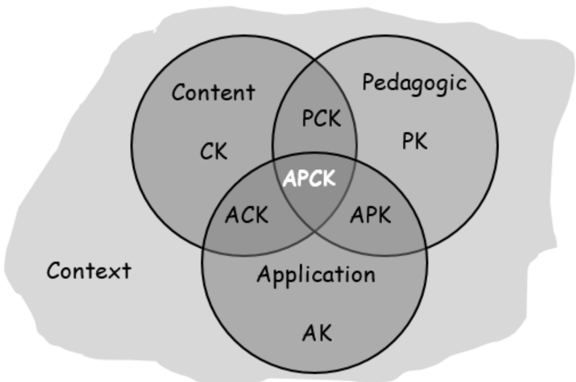


Figure 2. Application and Pedagogical Content Knowledge - APCK

On this model, Application and Content Knowledge (ACK) is directly linked to how the use of mathematical applications and the content influence each other. The use of mathematical applications promotes a different use of the mathematical knowledge, where the topics no longer are approached in a specific organized way. When working on mathematical applications all the mathematical knowledge of the students can be used at any time. As a result, the mathematics content no longer is addresses in a compartmented way. On the contrary, all the students' mathematics knowledge can be useful all the time.

The consequence is an impact from the use of applications on the mathematical content. In the same way, the Application Pedagogical Knowledge (APK), is directly linked to how the use of mathematical applications and the pedagogical approach influence each other. The use of mathematical applications requires a different approach, where options such as collaborative work, discussion of different approaches and presentation of the work developed are central.

Mathematical applications imply some real context and some interdisciplinary related knowledge. As so, Application Knowledge includes knowledge of disciplines beyond Mathematics, and this knowledge is not necessarily mastered by the teacher. The need to look for additional knowledge on fields outside mathematics requires a different attitude, appealing namely to reflection and critical reasoning. Reflection and critical reason are important skills for all mathematics' students and can be developed without using mathematical applications, the point is that applications turn them in central skills. In addition, once again, applications impact how the students learn mathematics (APK) and how the students come to think about what mathematics is (ACK).

Methodology

The study adopted a qualitative and interpretative methodology (Bryman, 2004). The data for the two case studies were collected by observation (two lessons) of a course of the master program for pre-service secondary school mathematics teachers and by an interview (after lessons). The analysis is based on the APCK model and intends to identify the domains of knowledge emphasized on each of the options assumed (based on the actions taken during the lessons and on the justifications presented for them on the interviews). As a consequence of the framework, a special attention is given to the mobilization of isolated or interlinked domains of knowledge. The analysis started from the identification of the reasons presented by the pre-service teachers for the options assumed and proceed relating these reasons to knowledge domains on APCK model.

A set of seven tasks, suitable for 10^{th} grade students, was given to the pre-service teachers: (1) The construction of the gutter, (2) Folding the corner of a sheet, (3) Slalom, (4) The trains, (5) Apples, (6) The colony of bacteria, (7) The box. Then, they were asked to choose three of these tasks and to develop lessons plans based on them. Their decisions should be guided by the potential they ascribe to each of the tasks to promote the students' exploration of a situation from reality. Afterwards, the pre-service teachers comment on the reasons that guided their choices, justifying why they choose (or not) each of the tasks. Being the criteria for choosing or excluding a task based on the way how it relates to reality, the tasks were chosen to include different circumstances, as justified on the next section.

Tasks and pre-service teachers' choices and approaches

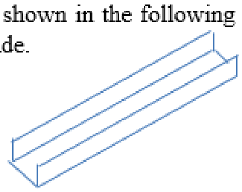
This section includes a brief description of the tasks and of the options of the group AB. This group includes two pre-service teachers, here designated by pre-service teacher A and pre-service teacher B, chosen because of their determination in their choices.

The three tasks chosen by this group from the set of seven tasks was: (2) Folding the corner of a sheet, (4) The trains, and (5) Apples.

'The construction of the gutter' was the 1^{st} task (see Figure 3), and it was included on the set of tasks because it was a very open task, clearly including a context from reality, where the students need to analyse the situation and take decisions about what to do. There is a need to think about the function of a gutter, calling for some interdisciplinarity between mathematics and engineering, or some application of mathematics on the field of engineering.

(1) The construction of the gutter

One company produces rectangular metal plates of 30 cm wide. Joining these plates, the company constructs gutters to be placed on the roofs of the houses. The plates are constructed by folding equal sidebands, which form right angles with the base, as shown in the following figure. Many different folds can be made.



Your job is to study the situation and propose to the company the best measure for the folding of the plates, presenting the reasons that led to your decision.

Suggestion: you can begin by analysing the capacity of the gutter in function of the folding that is made.

(Adapted from Neves, M., Guerreiro, L., Leite, A., & Silva, J., 2010, p.62)

Figure 3. Task 1: The construction of the gutter

This group did not choose task 1 (The construction of the gutter) because it was assumed as a not very well-defined task. According to the pre-service teachers: “We think this task is very vague about what the students are meant to do. So, we think they would have many difficulties and therefore we did not choose it.”.

The pre-service teachers think that the task is not clear enough about what the students are supposed to do and, consequently, they are afraid that the students can move away from what is intended and lose the focus on the mathematical content that they are supposed to work.

‘Folding the corner of a sheet’ was the 2nd task (see Figure 4), and it was included in the set of tasks because it requires data collection (students are asked to folder the sheet of paper and register some measures), however it cannot be considered exactly as a situation from reality or an application of mathematics.

(2) Folding the corner of a sheet

Fold a sheet of paper so that the upper left corner touches the underside of the sheet as shown in the figure.

Fold the sheet of paper in different ways and register the length of *a* and *x*. Analyse your data and find out what is the triangle (T) of larger area formed in the lower left corner of the sheet by the effect of this fold?
(consider a sheet of paper of 29 cm × 21 cm)

Figure 4. Task 2: Folding the corner of a sheet

The task 2 (Folding the corner of a sheet) was assumed as a valuable one. The main characteristic of this task emphasized by the pre-service teachers is the need to collect data. From the point of view of the pre-service teachers, this is very important because it stresses the connection between Mathematics and reality.

> "In our opinion this task, asking the students to collect data and using the analysis of these data to find the answer, shows how Mathematics has to do with reality. In addition, this is something that is very important to show to the students. Then, this situation corresponds to a polynomial function and so it is something that has everything to do with the Mathematics syllabus.".

This group also highlight the close connection to the syllabus allowed by this task, mentioning how it allows the work around a third-degree polynomial function. Another characteristic of this task, assumed as important by this group, is the possibility of using it to introduce the study of third degree polynomial functions or using it to deepen the students' knowledge on this kind of functions.

'Slalom' was the 3rd task (see Figure 5) included in the set of tasks. This is an open task with a sport context. As so, it can be assumed as an application of mathematics on sports, or as an interdisciplinary approach between mathematics and sports. However, it is mainly a mathematics task, being difficult to assume it has an application of mathematics. In fact, having the skier moving in a parabolic route does not seem to be a very real situation.

(3) Slalom

At a Slalom's competition, the skier must make a route passing between two flags forming a door.

1. Define the calculator window to $x \in [-1, 8]$, $y \in [-1, 7]$. Draw the points (1, 4), (2, 4), (5, 4), (6, 4), that will represent the Slalom flags.
2. Find a course for the skier, defined by a quadratic function and going through both doors without touching the flags.
3. The public attending the competition occupies the area in the plane defined by $y \geq 6$. Find a new quadratic trajectory for the skier so that he is not crossing the area reserved for the public.
4. The following illustration shows the path made by the Swiss champion who won this Slalom. Find the expression of a piecewise function with this graphic representation.

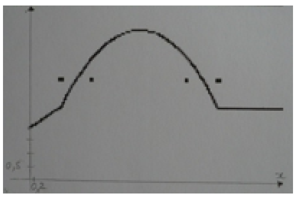


Figure 5. Task 3: Slalom

The task 3 (Slalom) was another of the tasks not chosen by this group of pre-service teachers. According to them and keeping in mind they were asked to choose tasks allowing the students to explore some situations from reality, they found that in this case the reality was somehow unreal. "We think this task is a bit artificial and has nothing to do with the students' reality. Still, the quadratic function can be worked on."

Nevertheless, the pre-service teachers address the fact that the task allows an approach to the quadratic function, one of the contents of the syllabus. They assume the task as a good opportunity for the students to look for suitable functions for the router of the skier, allowing the development of their knowledge on the impact of the parameters on the resulting parabola. As so, they recognize some value on it.

'The trains' was the 4th task (see Figure 6), and it was included in the set of tasks because it has obviously a real context. The task could be addressed in the mathematical field of operations research, however, it does not seem very important to get information about the moment and place where the trains cross. Also, the travelling speed of the trains seems to be a little bit unreal.

(4) The trains

The cities of Lisbon and Oporto are 315 km away from each other. A train departures from Lisbon to Oporto traveling at an average speed of 50 km/h. At the same time, another train departs from Oporto to Lisbon traveling at 40 km/h.
How much travel time does it take for the trains to cross?
How far from Lisbon do the trains cross?

(Adapted from Neves, M., Guerreiro, L., Leite, A., & Silva, J., 2010, p.50)

Figure 6. Task 4: The trains

The task 4 (The trains*)* was one of the tasks chosen by the pre-service teachers. The main reason presented to justify the preference for this task is related to a strong connection to reality. The reasoning required for the students to solve the task is another point addressed and a point that is assumed as an important one. "We think it is a task that has a lot to do with reality. And it is a problem that requires the students to think. We think it's a good task to introduce the linear function."

Despite the relevance ascribed to the reality on the situation presented, the reasonableness of the speeds of the trains considered are not taken into account. It is also not considered the interest of knowing where the trains cross. And even when confronted we these two points, the pre-service teachers keep their point of view unchanged, stating that this task has a strong connection with reality.

'Apples' was the 5th task (see Figure 7), and it was included in the set of tasks because it starts addressing the important issue of healthy eating. However, the task is about fruit flies. Even so, it can be assumed as an interdisciplinary approach between mathematics and biology, or an application of mathematics on the field of biology, but the model presented is not the best to model population grows or decrease.

(5) Apples

Healthy eating must be rich in fruit. Fruit is rich in vitamins, minerals and phytochemicals that usually have an antioxidant function, among other beneficial effects for health.
But sometimes plagues appear that attack the production of fruit. Last year an apple tree plantation was invaded by a plague of fruit flies. It was found that the number N of fruit flies, in thousands, evolved over time t, in days, according to the following law:
$N(t) = -t^2 + 18t + 40$

a) What was the initial number of fruit flies?

b) After how long has the plague been exterminated?

c) On what day were the largest number of fruit flies detected and what was the number?

(Adapted from Neves, M., Guerreiro, L., Leite, A., & Silva, J., 2010, p.75)

Figure 7. Task 5: Apples

The task 5 (Apples), was another of the tasks chosen by the pre-service teachers. In the case of this task, the most important issue for the decision to choose this task is related to the reference to healthy food. This is strongly valued by the pre-service teachers, although this reference is not truly linked to the task. In fact, the task is about a plague of fruit flies. Nevertheless, the initial reference on the task to the relevance of healthy food seems to be relevant enough for these pre-service teachers. They also address the work around the quadratic function as another important element on this task. "We think this task has something very important that is the reference to the issues of healthy food. It alerts for this. And then it allows the students to work on the quadratic function."

When discussing the connection to reality, it does not seem important to these pre-service teachers that the best model to describe a population increase or decrease is usually an exponential model and not a quadratic one (a potentially relevant point when the intention is to choose tasks to stress the connection between mathematics and reality).

'The colony of bacteria' was the 6th task (see Figure 8). This task is very similar to the previous one, including a model for the number of bacteria in a colony (instead of a model for the number of flies). The main difference is that in this case the task does not address any relevant issue, as it was the case on the previous task.

(6) The colony of bacteria

In a laboratory, a bacterial colony was studied. At eight o'clock, it was done the first counting and, after that, a new counting was done every hour. It has been found that the number N of bacteria, in thousands, after h hours, is given by $N(h) = -h^2 + 4h + 9$.

a) How many bacteria did the colony have at 8 o'clock?
b) What was the result of the second counting?
c) At what time of the day did the number of bacteria exceed 9000?
d) Describe the evolution of the colony from 8 to 13 hours.
e) At what time was the colony extinct?

(Adapted from Costa & Rodrigues, 2007, p. 118)

Figure 8. Task 6: The colony of bacteria

The similarity between task 6 (The colony of bacteria) and the previous task (the one about apples) is recognized in terms of mathematical content. As so, the pre-service teachers think it does not make sense to choose both. They think the situation on this task is not very appealing to the students and, on the contrary, the previous one addresses healthy food, a very relevant issue. Therefore, they choose task 5 and not this one.

> "This task is very similar to that of apples. As so it makes no sense to choose both. It also addresses the quadratic. We chose apples' task for its reference to healthy food. And also, because apples are more interesting than bacteria, are not they?"

'The box' was the 7th task (see Figure 9), and it was included in the set of tasks because it consists on a search for a solution for a situation intending the building of something. Although it is presented as a particular problem, it can be related to economic issues.

Similarly, to what happened on tasks 5 and 6, in the case of task 7 (The box), the first analysis is based on the mathematical content addressed. This task focus on a 3rd degree polynomial function.

(7) The box

Laura intends to build a box without a lid to store her brother's toys. For this, she has a rectangular card with 1.2 m long and 80 cm wide, where she intends to remove four square corners to facilitate folding the sides of the box.
What is the square side length that Laura should cut at every corner of the card to get a box of maximum volume?
(display the results in centimetres, rounding it to two decimal places)
Steps to follow in the resolution:
- Show that $V(x) = 4x^3 - 4x^2 + 0{,}96x$, being V the volume of the box.
- Explain the variation of x and use the graphing calculator to get a graphical representation of the function.
- Calculate x so that the volume of the box is maximum.

(Adapted from Costa & Rodrigues, 2010, p. 93)

Figure 9. Task 7: The box

That is also the case of the task 2 (Folding the corner of a sheet). As so, the pre-service teachers decided that only one of these tasks should be chosen. The fact that the task 2 address the same content, but requires real data collection, is considered potentially more interesting for the students and it is decisive for the pre-service teachers' choice.

> "It's a volume, so it's a 3rd degree polynomial function. And, well, we think the sheet of paper task is more interesting because it includes data collection. We think that it would make it more relevant for the students."

Conclusion

The main conclusions point to an appreciation of the mathematical part of the task and to a devaluation of the remaining components. The focus seems to be on the mathematical content addressed, suggesting a stronger emphasis on the Content Knowledge and a devaluation of the Pedagogic Knowledge. The way how reality appears on the tasks seems not to be assumed as very relevant, suggesting that ACK and APK are not very developed. This result in a choice of tasks where the answers do not make sense in a real context or where the mathematical model advanced on the task is not suitable to model that kind of situation. The interdisciplinary character of the situation (APCK) most of the times seems to be neglected, resulting in a choice of tasks where its presence is poor or resulting in approaches where that component is not explored. These options illustrate a strong appreciation of mathematical procedures and some tendency to approach the contents separately: there seems to be a strong appreciation of disciplinary knowledge, very marked by a traditional approach to the curriculum, where the disciplines and contents of each discipline are approached separately. Opportunities to create bridges to other disciplines studied by the students, to enlarge the students' culture or to promote their curiosity for new fields are not valued. The usefulness of mathematics in so many real-life situations is not valued either, although it could be used to promote the students' interest in mathematics. And the reason for that seems to be related to the pre-service teachers' knowledge. Understanding the knowledge needed to adopt practices that differ from the more traditional ones is a very relevant matter and somehow not a very well studied field. As so, it will be important to deepen the understanding about the way how pre-service teachers conceive the integration of applied tasks on the teaching and learning of Mathematics.

Acknowledgements

This work was supported by the Portuguese Science Foundation - FCT, through the project PTDC/CED-EDG/32422/2017.

References

Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special?. *Journal of Teacher Education*, *59*(5), 389-407.

Bergsten, C., & Frejd, P. (2019). Preparing pre-service mathematics teachers for STEM education: An analysis of lesson proposals. *ZDM Mathematics Education*, *51*(6), 941-953.

Bryman, A. (2004). *Social research methods*. Oxford: Oxford University Press.

Costa, B., & Rodrigues, E. (2010). *Novo Espaço - Parte 2, Matemática A 10.º ano* (pp. 93-118). Porto: Porto Editora.

Drake, S. (1991). How our team dissolved the boundaries. *Educational Leadership*, *49*(2), 20-22.

Ferri, R., & Mousoulides, N. (2017). Mathematical modelling as a prototype for interdisciplinary mathematics education? - Theoretical reflections. In T. Dooley & G. Gueudet (Eds.), *Proceedings of CERME 10* (pp. 900-907). Dublin, Ireland: ERME.

Fogarty, R. (1991). Ten ways to integrate curriculum. *Educational Leadership*, *49*(2), 61-65.

Hill, H., & Ball, D. (2009). The curious - and crucial - case of Mathematical Knowledge for Teaching. *Phi Delta Kappan*, *91*(2), 68-71.

Mishra, P., & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, *108*(6), 1017-1054.

Neves, M., Guerreiro, L., Leite, A., & Silva, J. (2010). *Matemática A - 10.º ano, Funções I* (pp. 50, 62, 75). Porto: Porto Editora.

Roth, W-M. (2014). Interdisciplinary approaches in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 647-650). Berlin, Heidelberg: Springer.

Ruthven, K. (2011). Conceptualising mathematical knowledge in teaching. In T. Rowland & K. Ruthven (Eds.), *Mathematics knowledge in teaching* (pp. 83-96). Dordrecht: Springer.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.

Williams, J., Roth, W.-M., Swanson, D., Doig, B., Groves, S., Omuvwie, M., Borromeo Ferri, R., & Mousoulides, N. (2016). *Interdisciplinary mathematics education - a state of the art*. Berlin: Springer.

L. Leite et al. (Eds.) (2019), Proceedings of the ATEE Winter Conference 'Science and mathematics education in the 21st century'
Brussels: ATEE and CIEd (ISBN: 9789082506549)

Mathematics trails: Opportunities for active learning in mathematics

Rosa Antónia Tomás Ferreira
Faculdade de Ciências e CMUP, Universidade do Porto, Portugal
rferreir@fc.up.pt

Abstract: Building on the ideas of 'learning outside the classroom' and 'active learning', in this exploratory study, students' reactions to a mathematics trail, considering three dimensions of active learning - students' physical, intellectual, and social engagement - and students' affective responses to the whole experience are analysed. A class of 22 Portuguese sixth graders completed a mathematics trail, supervised by their mathematics teacher and some parents. The social engagement dimension of active learning was quite visible and showed itself as a key to the success of the intellectual dimension, mainly to cope with the most cognitively demanding tasks of the trail. The limitations of this study point out avenues for further research, especially related to the design of the trail's tasks and the need for articulation between in and outside the classroom learning contexts in order to foster students' establishment of mathematical connections of diverse nature.

Keywords: active learning; learning outside the classroom; mathematics trails

Introduction

We live in an area of *boxed* children, because children literally live in a box, whether at home, in the car, or in the classroom! Furthermore, children live attached to *boxes,* such as smartphones, tablets, televisions or computers. It is urgent to change this status. Outdoor activities, which push children to interact with each other and with the natural environment, which challenge them to think and connect (mathematical) ideas, and which engage them in physical motion are fundamental (Fenoughty, 2002; Fernandes, 2019; Vale & Barbosa, 2018). A mathematics trail is an instructional strategy that combines these three aspects: physical movement, cognitive challenge, and social interaction, in a learning environment outside the classroom. In this paper, the results of an exploratory study aimed at analysing students' reactions to a mathematics trail, as well as their affective responses to the whole experience are presented. The main aspects of the theoretical underpinnings of this study - the ideas of learning outside the classroom and of active learning - are summarized, and a mathematics trail, and how it links to those theoretical ideas, is described.

Theoretical background

Learning outside the classroom

When we talk about 'learning outside the classroom', we think of contexts whose characteristics move away from the typical inner spaces of school buildings. We can think of outdoor spaces, inside or outside school contexts or we can think of closed environments outside school buildings. However, many terms other than 'learning outside the classroom' are associated with this idea, and consensus is yet to be achieved, even taking into account cultural differences across nations (Higgens & Nicol, 2002). According to Malone (2008), learning outside the classroom encompasses the "opportunities initiated by teachers and/or students to engage with alternative learning settings to complement and/or supplement the formal indoor classroom curricula" (p. 7). The author uses the expression *experiential learning* to emphasize that this process develops knowledge, skills, and attitudes, provided that it builds on consciously thinking about a "direct and active personal experience combined with reflection and feedback" (Malone, 2008, p. 8). In this paper, the expression 'learning outside the classroom' is used bearing in mind this need for personal engagement and reflection so that learning be meaningful and long-lasting (see also Ponte, 2014).

Several studies have suggested that the learning resulting from activities outside the classroom, almost always outdoor (in the open air), has a significant impact in the quality of students' lives (e.g., Rickinson et al., 2004; Malone, 2008; Davies et al., 2013). Some of the benefits of learning outside the classroom include greater motivation, curiosity, creativity, enjoyment, and achievement, as well as the meaningful application of problem-solving strategies and thinking skills, and the realization of mathematics as a transversal subject, which is present everywhere (National Centre for Excellence in the Teaching of Mathematics - NCETM). Waite and Rea (2007) summarize the benefits of learning outside the classroom in four key areas:

> "(1) fitness - bigger spaces mean bigger movements are possible so exercise increases physical fitness, coordination and wellbeing;
> (2) realise - the natural world provides a real life authenticity for learning and helps to embed decontextualized learning;
> (3) extend - subjects covered in class can be productively extended by trips which bring new insights and learning;
> (4) enthuse - novelty, fresh air, space." (Waite & Rea, 2007, p. 53).

As the Learning Outside the Classroom Manifesto (DfES, 2006) states "every young person should experience the world beyond the classroom as an essential part of learning and personal development, whatever their age, ability or circumstances" (p. i). In particular, leaving the classroom facilitates authentic (or experiential) learning, since "what we see, hear, taste, touch, smell, and do gives us six main 'pathways to learning'" (p. 3).

Active learning

Active learning can be "generally defined as any instructional method that engages students in the learning process (…) [requiring] students to do meaningful learning activities and think about what they are doing" (Prince, 2004, p. 223). We can find the echoes of this idea in several documents (e.g., NCTM, 2000, 2014; Ponte, 2014; Stein, Engle, Smith, & Hughes, 2008), especially regarding the need to involve students in challenging tasks of diverse nature, to engage them in working collaboratively and in discussing mathematical ideas, and to provide them with opportunities to reflect on what they do. Thus, active learning might be perceived as a different term for already existing and important ideas in the mathematics education arena. Yet, as explained next, some nuances help in better understanding the use of the term active learning in this paper.

Problem-solving tasks are an example of an active learning strategy. "They require more than the application of routine procedures, demanding students to make explicit, explain, and justify their reasoning" (Vale & Barbosa, 2018, p. 3), hence fostering students' intellectual engagement. But interaction is also important! Promoting students' engagement in social interactions is a fundamental role of the teacher, inside and outside the classroom. Social interactions - particularly through presenting, discussing, justifying, and connecting mathematical ideas and solution strategies - allow for sharing and developing mathematical meanings, but the teacher must ensure that students feel safe and confident in expressing their ideas and in taking risks (e.g., Vale & Barbosa, 2018).

Furthermore, students also need to move, i.e., to be physically active, inside and outside the classroom. Many studies emphasize the benefits of physical activities in several domains, including mathematics achievement (e.g., Marques, Santos, Hillman, & Sardinha, 2017, cited in Fernandes, 2019). Vale and Barbosa (2018) stress that students, especially the younger ones, need to be physically active, if not for physiological reasons, due to the long periods of inactivity and attention typical of traditional learning environments. Thus, adding movement to more usual problem-solving activities (e.g., based on worksheets) offers an example of an active learning strategy in the sense assumed in this paper.

Expanding on the work of Nesin (2012), Vale and Barbosa (2018) claim that:

> "actual *physical movement* helps students in focusing attention and in improving understanding and memorization; [*intellectual engagement* includes students involved in] investigating and finding relationships of diverse nature, which lead to the construction of meanings; [and] *social engagement* facilitates interaction, lends itself to collaborative work, and stresses the importance of active listening." (p. 3, italics added).

As such, in this paper, like Vale and Barbosa (2018), it is assumed that "learning emerges from the experiences and interactions amongst the intellectual, social, and physical dimensions" (p. 3) of the mathematical activities students engage with, as depicted in Figure 1.

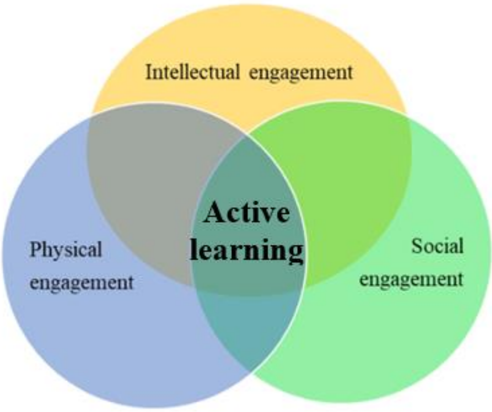


Figure 1. Dimensions of active learning (Vale & Barbosa, 2018)

Mathematics trails

A mathematics trail is, basically, a sequence of stops, along a predetermined path, in which students get involved with mathematics in the surrounding environment (Cross, 1997). Thus, they can use and apply, in real contexts, the mathematics that they have learned in the classroom, and they can mobilize their informal knowledge to deal with the various situations they find throughout the trail. This instructional strategy builds clearly on the notion of learning outside the classroom, and stimulates students' active learning of mathematics.

Children can make experiments and choices, take risks, solve and formulate problems, communicate mathematically, make connections, and appreciate the beauty and value of mathematics in their daily lives. Such direct experiences with the environment "become learning scenarios in real contexts, which give meaning to the [mathematical] concepts learned in the classroom, and involve students in active learning through exploration and discovery" (Tomás Ferreira, Vale, & Barbosa, 2016, p. 358; Cross, 1997). In addition, these outdoor learning experiences foster positive attitudes towards mathematics (e.g., Barbosa & Vale, 2016; Fernandes, 2019; Fernandes, Vale, & Palhares, 2017; Oliveira, 2018; Shoaf, Pollak, & Schneider, 2004).

The three dimensions of active learning (physical, social, and intellectual engagement) are clearly present in a mathematics trail, which, by design, provides a context for learning outside the classroom. A mathematics trail is obviously a learning experience that is movement-driven as students must go through the trail. Given that, a mathematics trail engages students in solving tasks of diverse nature and levels of cognitive demand (Shoaf et al., 2004), they are intellectually engaged with the various mathematical tasks they encounter along the trail. In addition, though a mathematics trail can be tracked individually, it is meant to be a group experience, thus promoting social interactions and exchange of ideas, approaches, and solutions to the tasks.

Besides offering a great potential for learning at all ages, mathematics trails may also be "valuable resources for teachers, given that they may facilitate access to evidences of students' understandings and difficulties" (Fernandes, 2019, p. 87). Additionally, students' work in the mathematics trails can serve as promising springboards for productive classroom collective discussions of relevant ideas and mathematical processes (Stein et al., 2008). In fact, experiences outside the classroom may contribute to support meaningful learning of mathematics "especially if the approaches in and outside the classroom are [adequately] articulated" (Fernandes et al., 2017, p. 204). Such an articulation helps students in developing a broader perspective about mathematical concepts, understanding them in a more relational way. In fact, "learning mathematics outside the classroom is not enrichment, it is at the core of empowering an individual's understanding of the subject" (NCETM).

Although mathematics is everywhere and, therefore, any place can work as the grounds for a mathematics trail, its construction is a challenging activity for teachers, especially concerning the choice/design of tasks that take full advantage of the surrounding environment and that represent a good mix of levels of cognitive demand (e.g., Barbosa & Vale, 2016; Tomás Ferreira et al., 2016). The stops of the trail must be selected after identifying natural, physical, cultural, architectural, and historical aspects of the environment that may work as interesting contexts for the mathematical tasks. To choose a path, one has to decide on the length of the trail, which, in turn, depends on how the distance is going to be tracked (on foot, by public transportation or a combination of both), the number of tasks to be completed, and the estimated time students will need to solve those tasks and complete the trail. Each stop of the trail may be the scenario for one or more tasks. Tasks should be diverse in nature, varying in levels of cognitive demand and addressing distinct mathematical topics (though all topics may pertain to a single thematic area, for example, geometry). The main issue here is that the students need to be at the scene in order to solve the tasks, which means that if they were inside a classroom, they had no sufficient information to solve them. A notebook and a toolkit (with pencils, measuring material, calculators…) are also necessary in most trails (Shoaf et al., 2004).

In Portugal, some researchers have studied the potentialities of mathematics trails as resources for teacher education, by challenging future teachers to construct mathematical trails, which they can use with their future students (e.g., Barbosa & Vale, 2016; Tomás Ferreira et al., 2016). The difficulties in constructing challenging tasks, diversified in nature and in content topics involved, and adequate to a mathematics trail have been particularly evident, especially in those future teachers aimed at higher grades (7th through

12th) who tend to focus their trails on procedural knowledge and low cognitive demand tasks. A few studies involving student teachers, especially for the lower grades, have been undertaken. For example, Oliveira (2018) investigated the contributions of a mathematical trail for promoting fifth graders' understanding of geometric concepts. Besides a significant involvement on the children's part, using what they learned in the classroom to cope with the challenges faced in real contexts, these experiences have also shown the potential of mathematics trails to develop positive attitudes towards mathematics.

Working directly with schoolchildren, Fernandes (2019) took a different approach. She looked to identify the potentialities of trails to a more effective learning of mathematics, addressing cognitive and affective dimensions of learning. Despite some initial difficulties in understanding the tasks and the whole trail context and despite working in a distracting environment - the outdoors - students (3rd graders) engaged with the tasks with at least the same enthusiasm as they typically worked in the classroom. They showed quite a significant responsibility to solve, discuss, and register their solutions to the tasks, being less dependent on the teacher, and mobilizing several skills related to problem-solving, decision-making, and collaboration. The teacher's role was fundamental, though, concerning not only the design of the trails, but also the unlock of impasse situations, the promotion of students' reflection on their own work, and the guidance of students' discussions (Fernandes, 2019).

Context and methodological procedures

Assuming mathematics trails as opportunities for engaging students in active learning, outside the classroom, of mathematics (Vale & Barbosa, 2018), students' reactions to a mathematics trail were analysed, something that would be a rather new learning situation for them. The term *students' reactions* was used in a broad sense, tying it to the three dimensions of active learning considered before. As such, search for evidences of students' physical, social, and intellectual engagement with the trail, as well as of students' affective responses to the whole experience was perfomed.

A sixth grade mathematics and science teacher (in Portugal, 5th and 6th grade mathematics teachers are also certified to teach sciences, and they often teach both subjects to the same classes) was challenged to engage one of her 2017/18 classes in a mathematics trail. She enjoyed innovative approaches to teaching mathematics, but she had some trouble in finding, in her school setting, the support she needed to implement some instructional strategies that would move away from typical teaching approaches. The school where she taught was located in Ovar, a mid-size city in northern Portugal, near the sea, with a significant industrial area and a population of about 30 000 residents.

The foot trail started at the school, heading to the city centre, and ended at the school again. There were five stops, and the students had to solve one task (often with more than one question) in each stop. The tasks addressed different topics of both 5th and 6th grade mathematics curriculum, and they were of distinct nature and cognitive demand, though never being too challenging (at least, this was my hope). The 22 students (aged 10-11 years-old) were divided into four groups with five or six elements each. The author, the teacher, and two voluntary parents ensured the students' supervision along the trail. Each group walked through the same stops but in different sequences of stops to avoid having too many students at the same site at the same time. Each group had a worksheet with the instructions and the tasks, and students could use the worksheet to make their records while solving the tasks. Students carried pencils or pens, and a smartphone per group, which could work as a handheld calculator, if necessary. However, the smartphone had a deliberate different role: the students were required to take a group selfie when they completed each task, on the site of its corresponding stop; sometimes, the selfie was the answer to the task itself! This is why the mathematical trail was named 'Selfie-Paper in Ovar'.

In this qualitative and exploratory study (Yin, 1993), data were collected through direct observation, students' written productions (registered on the worksheets) and selfies, and pictures. A very short questionnaire was administered at the end of the mathematics trail to collect some data on students' affective responses to this experience. They were asked to complete the following sentences: (1) "What I enjoyed the most in the selfie-paper was…"; (2) "What I enjoyed the least in the selfie-paper was…"; and (3) "If we were to do another selfie-paper, I would like to…". A single query was made to the teacher and the parents who supervised the trail, asking for their opinion regarding the whole experience in written form.

Students' reactions to the mathematics trail: Evidences of physical, social, and intellectual engagement

Students' enthusiasm about leaving the school building and doing something different was evident. Some, though, were a bit sceptical about how fun the activity would be, given that it would involve mathematics. Yet, the scepticism quickly faded away and some resistant students became the leaders of their groups.

Figure 2 illustrates students' physical and social engagement with the mathematics trail. For about two hours, students walked through several arteries of the city, sometimes solving problems (see Figures 3, 5, and 6 for examples of such tasks) or more procedural tasks (like computing the value of numerical expressions) to find the location of the following stop. As in Fernandes et al. (2017) study, although the conditions were not very favourable for taking notes or writing down the answers to the tasks, students found their ways to go around the obstacles in this regard, by improvising a *human* table (see second part of Figure 2), or by sitting on the ground to make calculations or record answers (see third part of Figure 6).

Figure 2. Students' physical and social engagement in the tasks of the mathematics trail

Teamwork was always visible, and sometimes students had to make an additional physical effort to realize their strategies. For example, in the third part of Figure 2, students were trying to measure the height of a wall that they were supposed to fill in with tiles, which should be similar to those found in the right part of the building (see last part of Figure 2); the problem also asked them to estimate the necessary quantity of tiles for paving the left wall. In truth, students did not have to spend so much effort as the one shown in Figure 2, since the wall of the right part of the building, with the same height as the one they were working on, was already covered with the tiles they were asked to use. Thus, instead of making actual measurements of the dimensions of the wall, students could have turned to their estimation skills and solve the task more easily. Nonetheless, they were given total freedom to choose the strategies they wanted to solve the tasks.

The trail did fulfil its purpose in intellectually engaging students with the mathematics they found present in the surrounding environment of the stops. The tasks were diverse in terms of cognitive demand and some of them were probably too demanding for several students; yet, they should have the necessary tools for tackling the tasks and the group context was expected to help them in overcoming emergent difficulties along the trail.

Nonetheless, even the simpler tasks, from my perspective, ended up challenging the students in some way. For example, one of the trail's tasks stimulated students' estimation skills related to costs of food items in a coffee shop (Figure 3).

> Go on 'Alexandre Herculano' Street, following the traffic direction until the coffee shop 'O Gaveto'. Stop there. Imagine that three of you wanted, each one, to have a drink and eat something here for a snack. Yet, all together, you only have 5,50€. One of you cannot eat or drink anything with sugar, and another does not like milk. Without making noise, get into the coffee shop and find out if the money you have is enough for what you want to consume. If so, present two possibilities for the snacks of the three of you (snacks must be different from person to person). Take a selfie at the door of coffee shop 'O Gaveto' to register the moment.

Figure 3. Task included in the mathematics trail about costs of food items

All groups but one had a similar initial reaction to this task: students started by choosing whatever items they preferred to eat or drink, respecting the task's restrictions. Yet, they quickly realized that their preferences were quite expensive! Therefore, they changed their approach, choosing a mixture of preferred (like a package of chocolate milk and a muffin) and much less expensive (like a small bottle of water and a chewing gum!) items. Figure 4 illustrates the work of a group of students on this task, as well as the group's answer, which was correct, despite the misspellings. Students listed two possibilities for the snacks, taking into account the food or drink restrictions imposed by the problem, and they computed the total cost of each set of items, which was under the maximum allowed of 5,50 €.

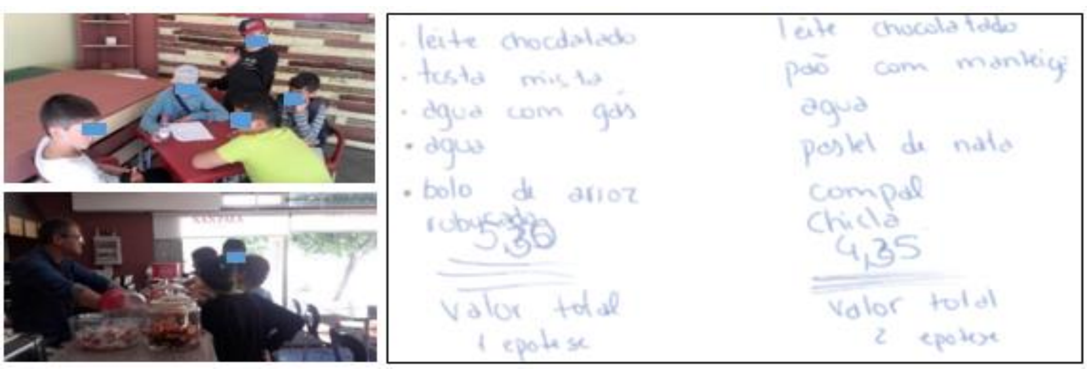


Figure 4. A group's answer to the task about costs of food items

Only one group answered the task in Figure 3 showing some sense of reality concerning the costs of food items typically available in local coffee shops. An apparently simple task (at least, it seemed so when it was designed) turned out being challenging to most students, who were not used to carrying money nor to thinking about the prices for the items they purchase at their school (at school, they carry a money card, which they only have to present in order to purchase the goodies they want; as such, they are not used to thinking about whether the *real* money they have is enough to buy the goodies - this information is given by the machine whenever they have insufficient funds). The shop's owner (who, of course, was already expecting for the students) had an unbeatable patience in responding to students' numerous queries since there was no visible menu, and even gave a candy to the children when they left his coffee shop!

There are many houses covered in tiles on both sides of the street where you are (…). Find a house whose tiles (search only for the little squares that form the tiles) have only one reflection of diagonal axis. Take a picture of the house's tiles to record your answer (you may take a selfie, as long as the tiles are clearly visible!)

Look for a house whose door number satisfies the following conditions: - it is a prime number; - it is smaller than l.c.m. (63, 27); - it is bigger than the quadruple of the sum of the square of 6 with the biggest composite number with one digit only. What geometrical figures do you find in the door of that house? Take a selfie in which the door number is visible in order to confirm your answer.

Figure 5. Students' collaborative work on tasks addressing geometric properties and tiles

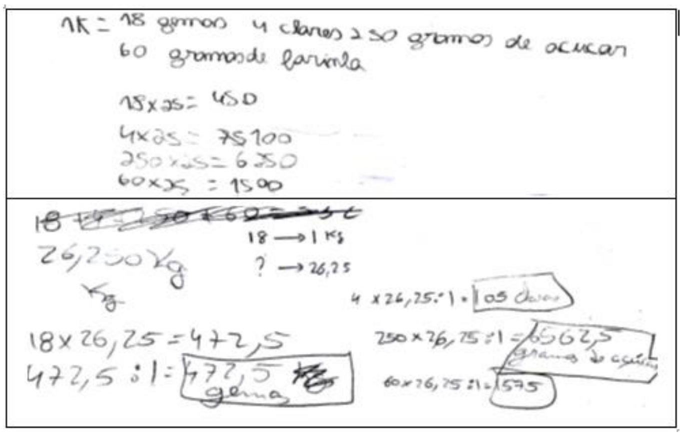
Ovar is known as the 'city tile (outdoor) museum'. Thus, inevitably, tiles were addressed in the trail's tasks, mainly focusing students' attention on some geometric properties in the little ceramics pieces that form the whole tile patterns. Figure 5 illustrates one of such tasks, in which students had to find out examples of tiles with only on reflection of diagonal axis. They had studied several types of rotations and reflections; for that reason, in other versions of the trail, students were asked to find houses whose tiles had other types of symmetry or no symmetry at all. The intention was that they could share and discuss their results in a classroom setting, but this, unfortunately, was not possible. Due to logistics constraints, the trail was tracked at the end of the school year only, and a collective discussion about the students' responses to the trail's tasks could not be made. Figure 5 also illustrates some evidences of students' social engagement in collaborative work, as well as one of the roles of smartphones in this 'Selfie-Paper in Ovar' - to take selfies!

Smartphones were also used to make computations (see the last part of Figure 6), though some students used a regular hand-held calculator (see the second part of Figure 2). In fact, students did not need very sophisticated materials to complete the trail, besides these devices and the given worksheet, pencils, and pens.

Keep on walking until you reach the *Casinha do pão-de-ló* store. Without making noise, get in and find the ingredients used for making the typical *pão-de-ló* of Ovar. List them. Investigate the quantities of each ingredient that are needed to make 35 *pães-de-ló* weighing 750g each.

Figure 6. Students working on the *pão-de-ló* task

The trail addressed other aspects of the local culture, such as the sweet gastronomic *ex-libris* of Ovar: *pão-de-ló*. Students were invited to quietly enter one of the many stores where this typical delicacy is sold and find out the ingredients necessary to make 35 *pães-de-ló* weighing 750g each. However, the *secret* recipe that was visible inside the store aimed at an estimated weight of 1kg (Figure 6).



1K = 18 gemas 4 claras 250 gramas de açucar
60 gramas de farinha
18x25 = 450
4x25 = 100
250x25 = 6250
60x25 = 1500

26,250 Kg
18 → 1 Kg
? → 26,25
18 x 26,25 = 472,5
472,5 : 1 = 472,5 gemas
4 x 26,25 : 1 = 105 claras
250 x 26,25 : 1 = 6562,5 gramas de açucar
60 x 26,25 : 1 = 1575

Figure 7. Students' solutions to the *pão-de-ló* task

This task was not anticipated as being very challenging; however, some groups did struggle to mobilize their proportional reasoning skills and answer correctly. For example, as shown in the upper part of Figure 7, students computed the quantities of ingredients necessary for making 25 *pães-de-ló* (their version of the trail asked for 25, not 35, *pães-de-ló*, which makes no difference in terms of the mathematics involved), but they did not take into account the weight of those sweets, which no longer was one kilogram. Another group of students (see bottom of Figure 7) calculated the total weight of 35 *pães-de-ló* and used that value (26,25 kg) to find the quantities of ingredients the task asked of them, resorting to the rule of three. Although the mathematical values they found were correct, their answer did not make complete sense, as it is not realistic to use 472,5 egg yolks (*gemas*) in a recipe!

As shown in Figures 5 and 6, several tasks directed students' attention towards cultural aspects of the city where their school was located. Not all students in this class lived in Ovar, but the trail allowed them, both the residents and the ones who lived in surrounding smaller communities, to look at many elements of the city with "mathematical eyes". As a side note, after the trail, a phone call from a mother was received, who asked what had been done to her son because he now walked on the streets and stopped at almost every house checking for the symmetries present in the houses' tiles! Realizing the presence of mathematics in places and aspects unlikely to cross students' minds was one of the goals of doing a mathematics trail (e.g., Cross, 1997; Shoaf et al., 2004). Despite being a first experience, the 'Selfie-Paper in Ovar' met this goal, launching the seeds for a different way of *seeing* the city.

The affective reactions to the 'Selfie-Paper in Ovar'

As in other studies (e.g., Fägerstam & Samuelsson, 2014; Fernandes, 2019; NCETM; Shoaf et al., 2004), the 'Selfie-Paper in Ovar' triggered students' positive affective responses. This is evident in their enthusiasm and engagement along the trail, but also in their answers to the questionnaire, which showed very positive reactions to the trail (Figure 8). The possibility of doing things collaboratively was the aspect students most emphasized positively about the trail. One might think that this was a rare group experience for these students, but this was not the case, as they had many class sessions in which they worked mathematics (and other subject areas) in a collaborative work setting. One might infer, thus, that the students emphasized collaborative work due to the context in which that work was carried out - the outdoors. This was, in fact, an innovative learning context for them.

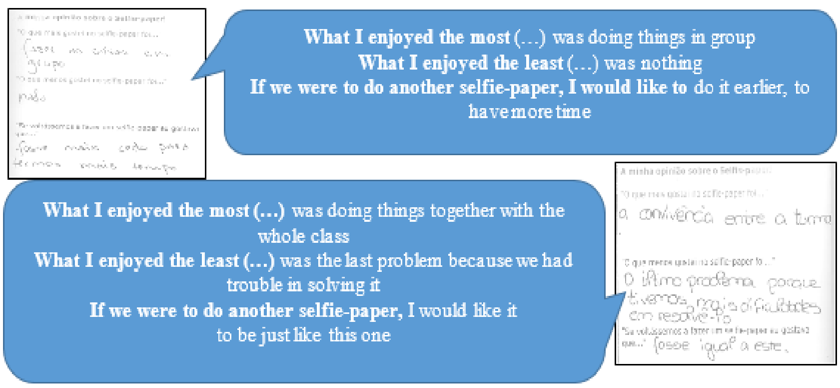


Figure 8. Examples of students' affective reactions to the trail

The aspects that caused students less positive reactions dealt with the (sometimes, excessive) cognitive demand of few tasks. In fact, the majority of the trail's tasks was accessible to students, but some of them were a bit too hard, and the collaborative setting was not enough to overcome some of the difficulties that emerged in some tasks. In addition, students suggested that the trail be tracked earlier in the school year (it was only possible to do the trail at the very end of the school year), and that they have more time to complete it.

When asked to make an overall assessment of the whole experience, the teacher stressed its social engagement dimension. She emphasized how students had been able to collectively overcome their difficulties in solving (especially) the (harder) tasks, and how they had mobilized their informal knowledge to cope with the challenges. The teacher also stressed students' flexibility in choosing strategies to solve the tasks, discussing them amongst each other and making changes when need, thus recognizing students' intellectual engagement with the mathematics involved in the trail:

> "I was quite happy to see that the students' difficulties were overcome by team spirit and by allowing themselves to use informal mathematical thinking, inviting each other to establish a plan in a natural way, discussing it, putting it into action, and continuously changing strategies [when needed], thus making mathematics more dynamic." (Teacher)

As expected, the parents involved in supervising the children during the trail also reacted very positively, emphasizing how this kind of experience helps children realize how mathematics is connected to their lives, allowing them "to look at the place they live in a different way, noticing things that would remain unnoticed otherwise". The social engagement dimension of the trail also captured the parents' attention, who stressed how it activated "team spirit and mutual aid to attain a common goal", which are learning experiences that are useful for life beyond school. One parent raised the question of the level of difficulty found in some tasks, causing trouble to the students and to her as well. She suggested that the parents-supervisors "could have been given some hints to, in turn, give the groups when needed, and [could have been] 'taught' about when and how to give those hints" to the students during the trail. These suggestions are certainly very valuable for future experiences involving parents playing the role of students' supervisors along a trail.

Final remarks

In general, students' reactions to the trail were very positive, whether regarding the three dimensions of active learning assumed in this study or considering the affective responses to the whole experience. The physical dimension was inherent to this kind of learning activity outside the classroom. The social engagement was, perhaps, the most visible aspect of the trail, showing itself as a key to the success of the intellectual engagement dimension, mainly in coping with the most cognitively demanding tasks of the trail. Social engagement was also the feature that the teacher and the parents most emphasized positively.

The author designed the tasks, but her inexperience in working with students of these ages and the teacher's time constraints in providing feedback about the tasks definitely accounted for having tasks that were too demanding for the students, an aspect that they recognized being what they had enjoyed the least in completing the trail. As mentioned before, this was an exploratory study; future experiences will need the teacher's involvement in designing the tasks and in their connection to the surroundings, taking the most of the real context to learning and appreciating mathematics more.

Despite students' mathematically interesting productions during the trail, it was not possible to follow-up this activity in a classroom setting, especially to collectively discuss different approaches to solving some of the trail's tasks. In fact, time constraints prevented this experience from being implemented earlier in the school term, which would have allowed time for collective discussions and, thus, to bridge the two learning environments, in and outside the classroom, as recommended by research and curricular orientations (e.g., DfES, 2006; Fernandes et al., 2017; NCTM, 2014). The available time did not allow for interviewing the students or the parents who supervised the trail. Future research must overcome these limitations and, besides improving the data collection procedures, include the articulation between in and outside the classroom learning contexts so that students can better connect mathematics to real contexts and daily life, making it more meaningful.

In this study, explicit ways the connections of mathematics and other school subjects, were not explored; only an initial approach to the links between mathematics and some cultural aspects of the city of Ovar were made. Yet, mathematics trails have great potential to help students establishing significant connections amongst different school subjects, for instance, sciences and arts. This is particularly relevant considering the current curricular challenges in Portugal, which call for a more integrated approach to teaching and learning all school subjects, emphasizing connections in a broad sense, in order to make learning more meaningful to all students. Further studies on the potentialities of mathematics trails can also provide valuable information for addressing active learning (of mathematics) in teacher education settings.

Acknowledgements

This work was partially supported by CMUP under Grant UID/MAT/00144/2019, which is funded by FCT with national (MCTES) and European structural funds through the programs FEDER, under the partnership agreement PT2020.

References

Barbosa, A., & Vale, I. (2016). Math trails: Meaningful mathematics outside the classroom with pre-service teachers. *Journal of the European Teacher Education Network, 11,* 63-72.

Cross, R. (1997). Developing maths trails. *Mathematics Teaching, 158,* 38-39.

Davies, D., Jindal-Snape, D., Collier, C., Digby, R., Hay, P., & Howe, A. (2013). Creative learning environments in education: A systematic literature review. *Thinking Skills and Creativity, 8,* 80-91.

Department for Education and Skills (DfES) (2006*). Learning outside the classroom manifesto.* Nottingham, UK: Department for Education and Skills.

Fägerstam, E., & Samuelsson, J. (2014). Learning arithmetic outdoors in junior high school - Influence on performance and self-regulating skills. *Education 3-13: International Journal of Primary, Elementary and Early Years Education, 42*(4), 419-431.

Fenoughty, S. (2002). The landscape of the school ground. In M. Hensle, P. Higgins, & R. Nicol (Eds.), *Outdoor education: Authentic learning in the context of landscapes* (pp. 25-28). Kisa: Kisa-Tryckeriet AB. Retrieved from http://www.docs.hss.ed.ac.uk /education/ outdoored/oe_authentic_learning.pdf

Fernandes, F. (2019). *A resolução de tarefas matemáticas em contextos não formais de aprendizagem - Um estudo com o 3º ano de escolaridade* (Doctoral dissertation). Retrieved from http://repositorium.sdum.uminho.pt/handle/1822/61132

Fernandes, F., Vale, I., & Palhares, P. (2017, April). A resolução de tarefas matemáticas em contextos não formais por alunos do 1º CEB. *Atas do XXVIII Seminário de Investigação em Educação Matemática*, 202-216.

Higgens, P., & Nicol, R. (2002). *Outdoor education: Authentic learning in the context of landscapes (Volume 2)*. Kisa, Sweden: Kinda Education Centre. Retrieved from http://www.docs.hss.ed.ac.uk/education/outdoored/oe_authentic_learning.pdf

Malone, K. (2008*). Every experience matters: An evidence based research report on the role of learning outside the classroom for children's whole development from birth to eighteen years.* Report commissioned by Farming and Countryside Education for UK Department of Children, School and Families. Wollongong, Australia. Retrieved from http://attitudematters.org/documents/Every%20Experience%20Matters.pdf

Marques, A., Santos, D., Hillman, C., & Sardinha, L. (2017). *How does academic* *achievement relate to cardiorespiratory fitness, self-reported physical activity and objectively reported physical activity: A systematic review in children and adolescents aged 6-18 years.* Retrieved from https://www.ncbi.nlm.nih.gov/pubmed/29032365

NCETM. *Learning maths outside the classroom*. Retrieved from https://webarchive.nationalarchives.gov.uk/20130323072419/https://www.education.gov.uk/publications/eOrderingDownload/LOtC.pdf

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM) (2014). *Principles to action: Ensuring mathematical success for all.* Reston, VA: NCTM.

Nesin, G. (2012). Active learning. In AMLE (Org.), *This we believe in action: Implementing successful middle level schools* (pp. 17-27). Westerville, OH: Association for Middle Level Education (AMLE).

Oliveira, A. (2018). *A Aprendizagem para além da sala de aula: Um trilho matemático no 5º ano de escolaridade* (Masters Dissertation). Escola Superior de Educação de Viana do Castelo. Retrieved from http://hdl.handle.net/20.500.11960/2025

Ponte, J. (2014). Tarefas no ensino e na aprendizagem da matemática. In J. Ponte (Ed.), *Práticas profissionais de professores de matemática* (pp. 11-27). Lisbon: Institute of Education, University of Lisbon.

Prince, M. (2004). Does active learning work? A review of the research. *Journal of Engineering Education, 93,* 223-231.

Rickinson, M., Dillon, J., Teamey, K., Morris, M., Young Choi, M., Sanders, D., & Benefield, P. (2004). *A review of research on outdoor learning*. London: National Foundation for Educational Research and King's College London. Retrieved from https://www.academia.edu/288162/A_Review_of_Research_on_Outdoor_Learning

Shoaf, M., Pollak, H., & Schneider, J. (2004). *Math trails*. Bedford, MA: COMAP. Retrieved from http://www.comap.com/highschool/projects/mathtrails/MathTrails.pdf

Stein, M., Engle, R., Smith, M., & Hughes, E. (2008). Orchestrating productive mathematical discussions: Helping teachers learn to better incorporate student thinking. *Mathematical Thinking and Learning*, *10*(4), 313-340.

Tomás Ferreira, R., Vale, I., & Barbosa, A. (2016, November). Trilhos matemáticos: Um recurso a explorar na formação inicial de professores. *Atas do EIEM2016, Encontro de Investigação em Educação Matemática*, 355-368.

Vale, I., & Barbosa, A. (2018). O contributo de uma *Gallery Walk* para promover a comunicação matemática. *Educação e Matemática*, *149-150*, 2-8.

Waite, S., & Rea, T. (2007). Enjoying teaching and learning outside the classroom. In D. Hayes, (Ed.), *Joyful teaching and learning in the primary school* (pp. 52-62). Exeter, UK: Learning Matters.

Yin, R. (1993). *Applications of case study research.* Beverly Hills, CA: Sage Publishing.

Solving combinatorial problems in lower grades of primary school

Snežana Gordić, Marina Milošević & Kristina Sabo
Faculty of Education in Sombor, University of Novi Sad, Serbia
snezana.gordic@pef.uns.ac.rs | marina.milosevic@pef.uns.ac.rs | pef.sabo@gmail.com

Abstract: Combinatorics develops logical thinking and reasoning, enhances problem-solving skills; combinatorial tasks often represent real-life situations, and therefore solving them improves pupils' functional knowledge. Mathematical curriculum for a primary school in Serbia does not contain combinatorics, and only some type of combinatorial problems appears in textbooks and within school competitions. Some literature suggested that children in early age can understand and solve different combinatorial tasks. Based on theoretical investigations we conducted a survey in lower grades of primary schools. The main goal was to determine whether pupils in lower grades of primary school are able to solve the combinatorial task without introduction and previous training. Results indicate that it would be good for mathematical education to include combinatorial problems and tasks in regular classes. It would also contribute to the development of pupils' interest in mathematics.

Keywords: combinatorics; level of difficulty of a task; logical thinking; mathematical achievements

Introduction

Combinatorics is an important field of mathematics that is widely used (in computer science, in industry, in business, in biology, in chemistry, in physics, in daily life) and it is today part of the mathematical curriculum for primary and secondary schools in many countries. The reasons for including combinatorics in schools are related to its usefulness in real-life and its pedagogical power. Combinatorics develops pupils' skills in reasoning and problem solving. It requires the creative work of the pupil and develops creative problem solving. Furthermore, combinatorics develops some ways of thinking, such as logical thinking, creative thinking and recursive thinking. Pupils' insight that combinatorics can be applied outside of school, in solving real-life problems, stimulates their motivation and activity. According to English (2005), "combinatorial problems also facilitate the development of enumeration processes, as well as conjectures, generalizations, and systematic thinking." (p. 122). Combinatorics improves pupils' functional knowledge. It is well known that functional knowledge should be developed from an early age, and therefore some combinatorial topics may be studied at early ages. English (2005) noted that "children should be given opportunities to explore combinatorial problem situations without direct instruction." (p. 134).

The fundamental aim of mathematical education is to develop pupils' skills in reasoning and thinking, enable problem solving and application of mathematical knowledge in everyday life. Combinatorics is a field of mathematics that may help pupils to achieve this aim, but it is not part of the mathematical curriculum for primary schools in Serbia. Some types of combinatorial problems appear in certain textbooks for primary school. In addition, combinatorial problems appear within mathematical competitions in lower grades of primary school. Here are some examples: 1) The combination for opening a safe is a three-digit number made up of different digits. How many different combinations can you make using only digits 1, 3 and 5? (Kengur bez granica, 2007); 2)

Janko has got four toys: a car, a doll, a ball and a ship. He wants to put them in a new order on the shelf. The ship must be next to the car, and the doll should also be next to the car. In how many different orders can he put the toys on the shelf? (Kengur bez granica, 2015).

This paper aims to determine whether pupils in lower grades of primary school can solve combinatorial problems without introduction and previous training. The second aim of this paper is to examine whether students enjoy solving combinatorial problems.

The plan of the paper is the following. In Section 2 we give a theoretical framework about learning and teaching combinatorics in lower grades of primary school, gender differences in learning mathematics. In Section 3 we pose our hypothesis and we describe instrument which is used. In Section 4 we presented obtained results. In Section 5 we discuss obtained results, state our conclusions and list open questions for further researches.

Theoretical framework

In this section, we present a brief overview of researches concerning teaching combinatorics in lower grades of primary school and the purpose of learning contents from combinatorics in lower grades of primary school. In addition, we provide an overview of researches on gender differences in learning mathematics.

Combinatorial reasoning, combinatorial problem solving and combinatorics in the mathematical curriculum

According to Piaget and Inhelder (1975), combinatorial reasoning plays a central role in the theory of cognitive development. Piaget carried out experiments that suggested that children can generate combinations in ages from nine to 11 but without a systematic procedure. Combinatorial reasoning is important for the development of early probability ideas. Probability has been included in the primary school curriculum in the early ages in many countries, so combinatorial reasoning should be developed in the early ages. According to English (2005), "combinatorics has an important role to play in the elementary school mathematics curriculum and should go hand-in-hand with children's experiences in probability." (p. 138). Some studies show that children have difficulty with probability because they cannot construct outcomes of combinatorial type (Johnson et al., 1998; Jones et al., 1999).

For the pupils who have been unsuccessful with mathematics, combinatorics offers the possibility for success. For talented pupils, combinatorics offers the possibility of a challenge. Combinatorics the gives teacher a new way to bring mathematics closer to pupils.

Melušová and Šunderlík (2014) concluded that the teacher has to have the appropriate knowledge to organize the content and the teacher should be able to track the process of development for pupils. They organized the sessions about problem solving in combinatorics within the mathematical module.

In 1991, English investigated the strategies that young children (in ages from four to nine) use in solving combinatorial problems on the different difficulty level. Krekić-Pinter, Ivanović, Namestovski, and Major (2015) defined strategies and methods for solving combinatorial problems (permutations, combinations and variations) in initial teaching of mathematics. They emphasize that pupils in fourth grade of primary school, with appropriate methodological transformation, are able to solve some types of combinatorial tasks.

Krpec (2014) studied the combinatorial skills of the lower primary school pupils through organizing objects (pictures, letters, dominoes, and numbers). That research showed that seven-year-old pupils could find successful strategies in organizing objects. According to Krpec (2016), "pupils who have problems with organizing objects based on given criteria have problems solving exercises from combinatorics" (p.3594).

Oparnica, Sudžuković, and Zobenica (2016) have proposed a lesson model. Arranging elements in an array "and sequence of combinatorial tasks that can be easily incorporated into existing programs in Serbia. In addition, they considered correlations of permutations to other school subject in lower grades of primary school, such as physical education, nature and social studies, art and music.

Metacognition plays an important role in mathematics learning. It could be defined as "a knowledge about thought process and strategies in learning mathematics and self-regulation in the process of solving mathematical tasks." (Oparnica, Marić, & Mihajlović, 2017, p. 21). For more details on metacognition and its role in learning mathematics, see paper written by Oparnica et al. (2017) and references therein. Biryukov (2004) confirmed the importance of metacognition in solving combinatorial problems.

Semadeni (1984) analysed the role of action proofs in primary mathematics teaching on the example of combinatorics problem (The number of permutation of *n* different object.).

These studies suggest that combinatorics can be taught in the lower grades of primary school and that it would be useful to include combinatorics in the mathematics curriculum for primary school. There are countries that already did it. Below we mention some of them.

Principles and Standards for School Mathematics in the United States (NCTM, 2000) recommend including discrete mathematics (including combinatorics, vertex-edge graphs, iteration, and recursions) in the curriculum for all grades, from pre-kindergarten through grade 12. In book 'Navigating Through Discrete Mathematics in Pre-kindergarten to grade 12', the authors provide content recommendations and classroom units for each grade. In pre-kindergarten - second grade, all pupils should sort, organize and count small numbers of objects; informally use the addition principle of counting; list all possibilities in counting situations; sort, organize, and count object using Venn diagrams. By the end of fifth grade, all pupils should represent, analyse, and solve a variety of counting problems by using arrays, systematic lists, tree diagrams, and Venn diagrams; use and explain the addition principle of counting; informally use the multiplication principle of counting; understand and describe relationships among arrays, systematic lists, tree diagrams, and the multiplication principle of counting. DeBellis and Rosenstein (2004) described the program that was developed to train teacher how to implement discrete mathematics in their classrooms.

In addition, the study of combinatorics begins in lower grades in some European countries. In Slovenia, students in lower grades of primary school collect and present combinatorial data, explore combinatorial situations and develop different methods for solving combinatorial problems (Program osnovna šola Matematika, 2011). In Slovakia combinatorics became part of the mathematical curriculum in 2008. From first to fourth grade pupils solve introductory problems in combinatorics and probability; from fifth to ninth grade pupils acquire skills in sorting objects according to given criteria through games and practical exercises, extract from a given group of elements a subgroup consisting of a given number of elements according to a given rule and calculate the number of possible selections (TIMSS 2015 Encyclopaedia). In Germany, pupils name the number of different possibilities in simple combination tasks up to fourth grade (TIMSS 2015 Encyclopaedia).

Gender-related differences in mathematics

Gender and personality variables related to attitude toward and ability in mathematics are important topics. Lachance and Mazzocco (2006) noted that "sex differences in math, on standardized tasks similar to those seen in school for primary school age children, are minimal or nonexistent" (p. 15).

In 1984, De Hernandez, Marek, and Renner investigated the relationships among gender, age and intellectual development. De Hernandez et al. (1984) tested 140 students, 70 males and 70 females, divided in the low-age group (age from 16.25 to 16.75 years) and the high-age group (age from 16.75 to 17.25), and found that "no gender differences exist in the ability to use combinatorial reasoning" (p. 374).

Flexer and Roberge (1980) concluded that boys in sixth, seventh and eighth grade have been shown to outscore girls in combinatorial reasoning.

Methodology

The problem, objectives and hypotheses

As a first objective of the research, we have to determine whether pupils in lower grades of primary school, i.e. in ages from seven to ten, are able to solve the combinatorial task *without* introduction and previous training. To address this question we have used a combinatorial task on permutations. We would like to find mean value, extremes and variations for the number of the written permutations. The ability for solving task was measured by the total points on the test. We would like to compare results on the ability for solving task between different groups of pupils: different age, male and female, different schools.

As a second object of the research, we have to examine whether pupils enjoy solving combinatorial problems. Our hypothesis is that pupils enjoy and like solving problems in combinatorics.

Results of the research will test the following hypotheses:

(1) Pupils in ages nine and ten have better ability for solving combinatorial task than pupils in ages seven and eight.
(2) In relation to gender, there is no difference in achievements in solving combinatorial task.
(3) In relation to the school, there is no difference in achievements in solving combinatorial task.
(4) More than 70% of pupils like this combinatorial task.
(5) More than 60% of pupils would like to solve tasks such as this one in the future.

Obtained data were analysed by the IBM SPSS Statistics (Version 20).

Sample

For the sample, we take 160 pupils in ages from seven to ten from different schools in the Province of Vojvodina in Serbia. The research included primary school "Avram Mrazović" in Sombor and primary school "Vuk Karadžić" in Novi Sad. The sample consisted of 121 pupils from the primary school in Sombor and 39 pupils from the primary school in Novi Sad. We have 42 pupils in age seven (or in the first grade), 41 pupils in age eight (or in the second grade), 36 pupils in age nine (or in the third grade) and 41 pupils in age ten (or in the fourth grade). In the sample, there were 84 male pupils and 76 female pupils.

Instrument

The instrument has two parts. The first part of the instrument is a test i.e. a complex combinatorial task (Table 1). We took the task from Oparnica, Sudžuković, and Zobenica (2016, p. 145) which contains several questions (on different difficulty level) concerning arranging elements from a given set. The question a) in the task is on the first difficulty level. The questions b) and c) are on the second difficulty level. The question d) is on the third difficulty level. This combinatorial task is easily understandable and put in a real-life context. For all ages, the test is the same.

Table 1. Combinatorial task

Filip, Nikola and Marko are jumping off the high diving board into the pool.	
Tasks:	**Solutions:**
a) What are the possible ways for these three boys to jump into the pool?	FNM NMF MFN FMN NFM MNF
b) What are the possible arrangements if Marko wants to jump before Nikola?	MNF FMN MFN
c) What are the possible orders if Filip does not want to jump first?	NMF MNF NFM MFN
d) If Oliver comes and wants to join his friends what are possible orders of jumping?	ONMF NOMF MONF FONM ONFM NOFM MOFN FOMN OMNF NMOF MNOF FNOM OMFN NMFO MNFO FNMO OFMN NFOM MFON FMON OFNM NFMO MFNO FMNO

The second part of the instrument is a questionnaire that consists of the following three questions: (1) Do you like math?; (2) Do you like this task?; (3) Would you like to solve more tasks like this in the future?.

The pupils should round up the answer YES or NO.

Results

First, we consider 'the number of written permutations' in each part of the combinatorial task. In the part a) of the combinatorial task, 20.6% of pupils listed all six permutations. In part b) 18.1% of pupils listed all possible permutations. In part c) 20% of pupils listed all possible permutations. In part d), which is at the third difficulty level, only five of 160 pupils, which is 3.1% of pupils, listed all possible permutations. Main descriptive statistics on the number of written permutation by parts of the combinatorial task are given in (Table 2). Results show that pupils are able to solve the combinatorial task without introduction and previous training.

Table 2. Descriptive statistics: The number of written permutations

Statistics	**Part a)**	**Part b)**	**Part c)**	**Part d)**	**Total**
Mean	2.81	1.39	1.79	4.40	10.39
Median	3.00	1.00	1.00	1.00	8.00
Mode	1	1	1	1	4
Std. Deviation	2.084	1.022	1.398	5.944	9.468
Minimum	0	0	0	0	0
Maximum	6	3	4	24	24

Next, we consider pupils' scores on the test. The score on the test is considered as a number of points won on the test. The part a) of the combinatorial task is scored with six points, the part b) is scored with three points, the part c) is scored with four points and the part d) is scored with six points. Maximal score on the test is 19 (six points at the first difficulty level, seven points at the second difficulty level and six points at the third difficulty level). Main descriptive statistics on the test scores are as follows: Mean value for all pupils is Mean=7.12, with the standard deviation SD=5.39. The most frequent score is Mod=3.25, and the median value is Med=6. The maximal score (19 points) is achieved by four pupils from Sombor and all of them are nine years old. These students are all from the same class and they did similar combinatorial tasks with the teacher in additional classes. The minimal score (0 points) is achieved by nine pupils (two pupils from Novi Sad and seven pupils from Sombor).

Comparison of the scores on the test between different groups of pupils

The scores on the test by ages are presented in (Table 3). We tested mean differences and find that there is a statistically significant difference between pupils in ages seven and eight and pupils in ages nine and ten. Pupils in ages nine and ten are more successful in solving this combinatorial task ($t(120.594)=-7.25$, $p<0.05$).

Table 3. The scores on the test by ages

Pupils' ages	**N**	**Mean**	**Std. Deviation**	**Std. Error Mean**
7 and 8	83	4.49	3.36	.369
9 and 10	77	9.95	5.57	.655

The scores on the test by gender are presented in (Table 4). We tested mean differences and find that there is no statistically significant difference between male and female ($t(158)=-0.124$, $p>0.05$).

Table 4. The score on the test by gender

Gender	**N**	**Mean**	**Std. Deviation**	**Std. Error Mean**
Male	84	7.07	5.48	.598
Female	76	7.17	5.32	.611

The scores on the test by the schools are presented in (Table 5). We tested mean differences and find that there is statistically significant difference between school in Sombor and school in Novi Sad ($t(85.364)=4.996$, $p<0.05$).

Table 5. The score on the test by the schools

School	**N**	**Mean**	**Std. Deviation**	**Std. Error Mean**
in Sombor	121	8.11	5.41	.492
in Novi Sad	39	4.04	4.05	.648

Whether pupils enjoy solving this combinatorial task we examine by use hypothesis test about a population proportion (large samples). We obtained that at the significance level 1% more than 70% of pupils like this combinatorial task ($z = 4.31$). and at the significance level 1% more than 60% of pupils would like to solve tasks such this in the future ($z = 3.07$).

Conclusions and discussion

Based on the results of the study carried out, we conclude that:

(1) Pupils in ages nine and ten have better scores than pupils in ages seven and eight. Results of this research show that there is statistically significant difference in the scores between different age groups. Pupils from all grades are capable to work on this combinatorial task without previous training, and pupils at older ages show significantly better results.

(2) As we suppose, there is no significant difference in the scores on the test between boys and girls. See references for more details on the gender difference in mathematics learning (Li, 1999; Samuelsson & Samuelsson, 2016; Skaalvik & Rankin, 1994).

(3) There is significant differences in total points between schools in Sombor and Novi Sad. It is probably due to the influence of the teacher. We do not examined whether teachers work with students tasks of combinatorial type.

(4) Pupils enjoy solving combinatorial problems. The percentage of all pupils who like this combinatorial task is greater than 70%. The percentage of all pupils who would like to solve tasks such as this one in the future is greater than 60%.

We indicate that it would be good for mathematical education to include combinatorial problems in regular classes. Teaching mathematics tends to achieve logically combinatorial thinking at pupils, and therefore combinatorics should be studied at an early age. Simple combinatorial problems related to the possible arrangements, grouping or choice of subjects should be solved in lower grades of primary school. Solving such problems is challenging for pupils and developing their creativity. Pupils enjoy solving combinatorial tasks, so it would contribute to the development of pupils' interest in mathematics. The teacher can use new methods and techniques, such as problem solving, detection, assumption method, the use of graphs and various diagrams.

The main limitation of this research are the following:

(1) The sample could be more representative. In future research, it should be included more pupils, from different schools.

(2) Pupils should have minimal instructions on how to write permutations. Pupils write full names given in the combinatorial task (for example Filip, Nikola, Marko, Oliver instead of FNMO). Therefore, the space provided for solutions is filled after several permutations. Few pupils entered abbreviations. Pupils do not have the developed skills to use abbreviations and mathematical notations. Combinatorics can develop those skills at early age.

A few questions remain open and can be addressed in future work:

(1) More combinatorial tasks should be included in the instrument. Variation and combination, addition principle of counting, multiplication principle of counting and tree diagrams should be a part of the instrument.

(2) Examine the ability for solving games of combinatorial nature (for example the Hanoi tower) in lower grades of primary school.

(3) Four pupils had the maximum score on the test. Would the test be used to detect pupils talented in mathematics?

(4) Are prospective teachers and teachers ready to teach combinatorics in lower grades of primary school?

(5) How combinatorics can be integrated in the mathematics curriculum for lower grades of primary school? What should a good mathematics curriculum containing combinatorics look like?

Acknowledgements

We would like to thank Professor Ljubica Oparnica, who gave us the idea for this research, for helpful discussions.

References

Biryukov, P. (2004). Metacognitive aspects of solving combinatorics problems. *International Journal for Mathematics Teaching and Learning, 25*(1), 1-19. Retrived from http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.178.3238

DeBellis, V., & Rosenstein, J. (2004). Discrete mathematics in primary and secondary schools in the United States. *ZDM, 36*(2), 46-55. Retrieved from https://link.springer.com/content/pdf/10.1007/BF02655758.pdf

De Hernandez, L., Marek, E., & Renner, J. (1984). Relationships among gender, age and intellectual development. *Journal of Research in Science Teaching, 21*(4), 365-375.

English, L. (1991). Young children's combinatorics strategies. *Educational Studies in Mathematics, 22*(5), 451-474. Retrieved from https://www.jstor.org/stable/pdf/3482465

English, L. (2005). Combinatorics and the development of children's combinatorial reasoning. In G. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 121-141). Springer, Boston, MA.

Flexer, B., & Roberge, J. (1980). IQ, field dependence - independence, and the development of formal operational thought. *The Journal of General Psychology, 103*(2), 191-201.

Johnson, T., Jones, G., Thornton, C., Langrall, C., & Rous, A. (1998). Students' thinking and writing in the context of probability. *Written Communication*, *15*(2), 203-229.

Jones, G., Langrall, C., Thornton, C., & Mogill, A. (1999). Students' probabilistic thinking in instruction. *Journal for Research in Mathematics Education*, *30*(5), 487-519. Retrieved from https://www.jstor.org/stable/pdf/749771.pdf

Kengur bez granica (2007). Задаци са такмичења „Кенгур без граница". Retrieved from https://dms.rs/kengur/zadaci/

Kengur bez granica (2015). Задаци са такмичења „Кенгур без граница". Retrieved from https://dms.rs/kengur/zadaci/

Krekić-Pinter, V., Ivanović, J., Namestovski, Ž., & Major, L. (2015). Strategy and methods for solving combinatorial problems in initial instruction of mathematics. *International Journal of Modern Education Research, 2*(6), 77-87. Retrieved from http://www.aascit.org/journal/archive?journalId=910&issueId=9100206

Krpec, R. (2014). The development of combinatorial skills of the lower primary school pupils through organizing the sets of elements. *Acta Mathematica*, *17*, 117-123. Retrieved from http://www.km.fpv.ukf.sk/upload_publikacie/20140625_113350__1.pdf#page=103

Krpec, R. (2016, November). What effect has the ability of the organization objects to solve combinatorial tasks. *Proceedings of ICERI2016 Conferences,* 3594-3599. Retrieved from https://www.researchgate.net/publication/311364578

Lachance, J., & Mazzocco, M. (2006). A longitudinal analysis of sex differences in math and spatial skills in primary school age children. *Learning and Individual Differences, 16*(3), 195-216.

Li, Q. (1999). Teachers' beliefs and gender differences in mathematics: A review. *Educational Research*, *41*(1), 63-76.

Melušová, J., & Šunderlík, J. (2014, June) Pre-service teachers' problem posing in combinatorics. *Proceedings of international conference 12th Mathematics Conference in Nitra held under the auspices. Acta Mathematica*, 115-122. Retrieved from http://www.nmk.fpv.ukf.sk/2014/download/AM_17.pdf#page=115

NCTM. 2000. Principles and Standards for School Mathematics, NCTM, 2000. Retrieved from https://www.nctm.org/Standards-and-Positions/Principles-and-Standards/

Oparnica, L., Marić M., & Mihajlović, M. (2017, October) Assessing metacognition and self-regulated learning in prospective mathematics teachers in Serbia. *Proceedings of the 42nd ATEE Annual Conference, 19-34.* Retrieved from https://bib.irb.hr/datoteka/957258.ZBORNIK_-_Conference_proceedings_ATEE_2017_Annual_Conference.pdf#page=32

Oparnica, L., Sudžuković, S., & Zobenica, M. (2016). Permutations in lower grades of primary school. *Norma, 21*(1)*,* 137-149. Retrieved from http://www.pef.uns.ac.rs/index.php/2016-05-24-05-15-13

Piaget, J., & Inhelder, B. (1975). *The origin of the idea of chance in children.* London: Routledge & Kegan Paul.

Program osnovna šola Matematika, 2011, Republika Slovenija. Retrieved from http://mizs.arhiv-spletisc.gov.si/fileadmin/mizs.gov.si/pageuploads/podrocje/os/prenovljeni_UN/UN_matematika.pdf

Samuelsson, M., & Samuelsson, J. (2016). Gender differences in boys' and girls' perception of teaching and learning mathematics. *Open Review of Educational Research*, *3*(1), 18-34.

Semadeni, Z. (1984) Action proof's in primary mathematics teaching and in teacher training. *For the Learning of Mathematics, 4*(1), 32-34. Retrieved from https://www.jstor.org/stable/pdf/40247842.pdf

Skaalvik, E., & Rankin, R. (1994). Gender differences in mathematics and verbal achievement, self-perception and motivation. *British Journal of Educational Psychology*, *64*(3), 419-428.

TIMSS 2015 Encyclopedia. The mathematics curriculum in primary and lower secondary grades in Germany. Retrieved from http://timssandpirls.bc.edu/timss2015/encyclopedia/countries/germany/the-mathematics-curriculum-in-primary-and-lower-secondary-grades/

TIMSS 2015 Encyclopedia.The Mathematics Curriculum in Primary and Lower Secondary Grades in Slovak Repuplic. Retrieved from http://timssandpirls.bc.edu/timss2015/encyclopedia/countries/slovak-republic/the-mathematics-curriculum-in-primary-and-lower-secondary-grades/

The notion of function held by basic education pre-service teachers

Floriano Viseu[1], Paula Mendes Martins[1] & Helena Rocha[2]
[1]Universidade do Minho, Portugal & [2]Universidade NOVA de Lisboa, Portugal
fviseu@ie.uminho.pt | pmendes@math.uminho.pt | hcr@fct.unl.pt

Abstract: The current curricular guidelines for mathematics education in Portugal emphasize the relevance of working with different representations of functions to promote understanding. Given this relevance, we seek understanding about the notion of function held by 37 basic education pre-service teachers in their first year of a master's course. Data were collected through a task focusing on identifying functions in situations based on different representations. The content analysis technique was then adopted in the search for an understanding of the justifications given by the participants. The results achieved suggest it is easier for the pre-service teachers to identify examples that are not functions than examples that are functions. There is also a tendency for greater accuracy in the identification of examples expressed by tables than by algebraic expressions. The justifications presented show a notion of function as a relation between values of two non-empty sets, but without guaranteeing that this relation is single-valued.

Keywords: basic education; functions; preservice teachers

Introduction

The concept of function is one of the important concepts of Mathematics. According to the Portuguese syllabus, this concept is addressed in formal terms for the first time in the 7^{th} grade (age 12) and keeps being developed until the 12^{th} grade (age 17). However, the inherent characteristics of the concept of function are essential for the introduction of mathematical topics in the first two cycles of basic education. As examples we can consider the existence and uniqueness of the results of arithmetic operations studied at the 1^{st} cycle (grades 1-4), and the relationship between any geometric figure and its area, studied at the 2^{nd} cycle (grades 5-6).

As in most of the countries in the world, the Portuguese education system comprises twelve years before entering higher education. Of these years, the first nine correspond to basic education and the last three to secondary education. In basic education (consisting of three cycles: the first lasts four years - grades 1-4 -, has a unique responsible teacher and is also known as primary school; the second cycle lasts two years - grades 5-6 - and the third three years - grades 7-9), the mathematics' curriculum is the same for all students.

The current curricular guidelines suggest the use of different representations of a function (numerical, tabular, algebraic and graphical), assuming their relevance to the students' understanding. As the concept of function is complex, each representation offers an opportunity to understand what could not be understood in another representation. The connection between different representations creates a global vision, which is more than the union of the knowledge relative to each of the representations, and which allows the development of a deeper understanding. The relevance of the concept of function and the role of representations in their learning makes teachers' knowledge a very important issue. In this study, we seek understanding over the notion of function held by pre-service teachers in their first year of a master's degree in basic education.

Theoretical framework

The evolution of the notion of function and its integration in the school curriculum

The concept of function is seen as one of the most important in all mathematics (Ponte, 1992) and is one of most complex concepts not only of school mathematics but also at undergraduate level (Safuanov, 2015). The evolution of the concept of function goes back 4000 years (Kleiner, 2012) and there are many particular examples of functions that can be found throughout these years, such as counting, which implies a correspondence between a set of objects and a sequence of counting numbers; the four elementary arithmetical operations, which are functions of two variables; and the Babylonian tables of reciprocals, squares, square roots, cubic, and cubic roots (Ponte, 1992). However, the notion of function did not explicitly emerge until early in the eighteenth century. According to Kleiner (2012), this is due to two main reasons: lack of algebraic prerequisites and lack of motivation. For this author, a number of developments were fundamental to the rise of the function concept:

> "- extension of the concept of number to embrace real and (to some extent) even complex numbers (Bombelli, Stifel, et al.);
> - the creation of a symbolic algebra (Viète, Descartes, et al.);
> - the study of motion as a central problem of science (Kepler, Galileo, et al.);
> - the wedding of algebra and geometry (Fermat, Descartes, et al.)." (Kleiner, 2012, p. 104).

According to Ponte (1990), the "origin of the notion of function is confused with the beginnings of Infinitesimal Calculus" (p. 3). The term 'function' was used for the first time by Leibniz in his manuscripts of 1673 (Safuanov, 2015) to denote "the dependence on a curve of geometric quantities as subtangent and subnormal" (Ponte, 1990, p. 3). Also, the terms 'constant' and 'variable' were introduced by Leibniz (Safuanov, 2015).

In the correspondence between Leibniz and Bernoulli from 1694 to 1698, the term 'function' was adopted for the purpose of representing quantities dependent on some variable by means of an analytic expression (Ponte, 1990). The definition of function was first formulated by Bernoulli in 1718, when he considered a function of a certain variable as an amount that is a combination of that variable and constants (Kleiner, 2012). This definition was refined by Euler, a former student of Bernoulli, who replaced the term 'quantity' with 'analytic expression' in 1748 (Ponte, 1990). It was Euler who introduced the notation (x) for the concept of function in 1734 (Safuanov, 2015). The definition proposed by Euler led to several inconsistencies and limitations, since the same function can be represented by different analytical expressions, but it eventually came into force in the eighteenth and nineteenth centuries (Ponte, 1990).

The notion of function has evolved due to its association with the notions of continuity and serial development. One of these developments resulted from Fourier's work, which addressed problems of heat conduction in objects in which he considered body temperature to be a function of two variables (time and space). Fourier conjectured that for any function it would be possible to achieve trigonometric series development at an appropriate interval. This statement was not proved by Fourier, but by Dirichlet, who formulated sufficient conditions for the representability of a function by a Fourier series (Ponte, 1990). In 1837 Dirichlet "then separated the concept of function from its analytical representation, formulating it in terms of arbitrary correspondence between numerical sets" (Ponte, 1990, p. 4). Thus, a function would consist only of a correspondence between two variables, such that for all the value of the independent variable one and only one value of the dependent variable is associated (Ponte, 1990). It was with the development of Cantor's theory of sets that the notion of function come to include anything that was an arbitrary correspondence between any sets, numeric or not. From the notion of correspondence to the notion of relation (Ponte, 1990).

Functions and their representations

Different representations in the teaching and learning of mathematics

Representations are assumed as central for students' learning (NCTM, 2000) and are often used to emphasize important mathematical concepts (Mitchell, Charalambous, & Hill, 2014). Conceptualized as entities that symbolize or stand for other entities (Duval, 2006; Goldin & Kaput, 1996), different representations can elucidate different aspects of the concept. They can help the students who are trying to make sense of the concept, offering some support to organize their ideas and develop mental models of the concept (Mitchell, Charalambous, & Hill, 2014). Simultaneously, the use of different representations can also create the opportunity to consider student diversity, creating space for different ways of reasoning and different preferences (Dreher, Kuntze, & Lerman, 2016). Consequently, representations can make abstract concepts more accessible to the students (Flores, 2002) and foster the connection between procedures and concepts (NCTM, 2000). This is the main reason why working with different representations and the connections among them plays a key role for learners in building up conceptual knowledge in the mathematics classroom (Dreher, Kuntze, & Lerman, 2016). However, as emphasized by Rocha (2016), the mathematical learning does not take place automatically just because the students use different representations. The representations are not inherently transparent (Meira, 1998). Thus, the students need opportunities to reflect on their actions and the teacher's guidance to make connections between representations and underlying mathematical ideas (Stein & Bovalino, 2001). In addition, many times, the teachers only use one representation or do not articulate the different representations used (Nachlieli & Tabach, 2012). This is why Mitchell, Charalambous and Hill (2014) address the ability to teach with representations as a critical component of teaching mathematics well. Dreher, Kuntze and Lerman (2016) go further, highlighting the relevance of specific knowledge and views about using multiple representations and the need to pay attention to it in the professional development of pre-service teachers. After all, only the combination of different representations affords the development of a rich concept image (Tall, 1988) and this requires the teachers' ability to recognize that and to design rich mathematical activities (Dreher, Kuntze, & Lerman, 2016).

Different representations in the teaching and learning of functions

The different representations of mathematical concepts are of great importance in student learning (Viseu, Fernandes, & Martins, 2017), because each of these representations adds or highlights something that is hidden or not prominent in other representations. Thus, the exploration of the different representations in learning is a requirement for a deeper understanding.

In the case of the concept of function, the main representations are the numerical, tabular, graphical and algebraic representations, each of which reveals specific aspects and properties of functions.

Numeric and tabular representations. These representations, which some authors (such as Cuoco, 2001, and Rocha, 2016) consider as distinct and others (such as Goos and Benninson, 2008, and Lesser, 2001) as the same representation, are based on one or more pairs of values of the variables involved in functional relationship. When several pairs of values are presented, the representation facilitates generalization, that is, facilitates discovery of a law of formation, which is characteristic of algebraic representation. In this representation, verifying that we have a function requires the student to analyse the numerical values that are given in the table, according to the relation in question. For Brown and Mehilos (2010), tabular representation facilitates the passage from concrete to abstract, giving meaning to algebraic variables and expressions.

Graphical representation. This representation consists of representing all the points the coordinates of which satisfy the functional relation in question. Compared with other representations, it reveals certain properties of functions, such as zeros, its sign, monotony, etc. In terms of finding whether a relation is a function, the criterion that any vertical line intercepts the graph in at most one point is adopted. According to Friedlander and Tabach (2001), the graphical representation is intuitive and appealing owing to its visual character.

Algebraic representation. This representation, in addition to the sets involved in the function, involves an algebraic relationship between the variables considered in the function. It is a highly compact and abstract representation in which the algebraic law plays a fundamental role in the study of the function, requiring the student to manipulate the relation algebraically to study function properties, including verifying that it is a function. For Friedlander and Tabach (2001), algebraic representation is precise, general and effective in the presentation of mathematical patterns and models and is often the only way to justify general statements.

Research methodology

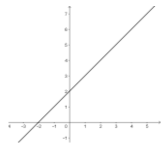
With the main goal of ascertaining the notion of function held by future basic education teachers, we proposed to 37 pre-service teachers, a task with relations between two variables defined through graphs, tables and algebraic expressions that should be classified as functions or not functions. The data were collected by one of the authors of this work, in the academic year 2018/2019, during one of the courses taken by the pre-service teachers. Some of the participants in this study were attending the first year of the master's course in pre-school and basic education (primary school) (M1, $n = 23$) and the others attending the master's course in 1st cycle of basic teaching and Natural Sciences and Mathematics teaching in the 2nd cycle of basic teaching (M2, $n = 14$).

Usually, students who apply to these two master's courses, offered by the same university, have a Basic Education Undergraduate degree. In terms of their learning process, every pre-service teacher studied mathematics at least up to the 9th grade, where they learned the topic of functions using their different representations. More specifically, in the 7th grade, the notion of function is introduced for the first time, the linear function is studied in the 8th grade, and the inverse proportionality and quadratic function of type $f(x) = ax^2,\ a \neq 0$ are studied in the 9th grade. In the task, eight items were proposed to the pre-service teachers, in which they were asked to identify whether each example represents a function or not (Figure 1).

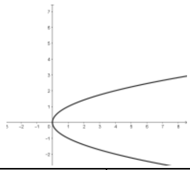
In the task proposed, items a), b), d) and g) involve the representation of relations through graphs; items c) and e) involve the representation of relations through tables; and items f) and h) involve the representation of relations through algebraic expressions. For each of these items, the 37 pre-service teachers were asked to identify if the example represented or not a function, justifying their answer.

Determine whether each relation is a function. Justify your answer.

***a*)**



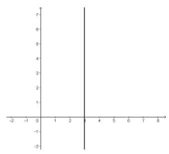
***b*)**



***c*)**

x	y
1	2
2	3
3	2
4	5

***d*)**

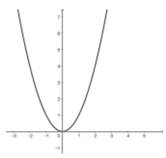


***e*)**

x	y
1	3
1	4
2	6
3	5

***f*)** $x = -2$

***g*)**



***h*)** $y = x^2 - 2x$

Figure 1. Task proposed to the preservice teachers

The answers given by the pre-service teachers were classified as correct (C), partially correct (PC) or incorrect (I). An answer was considered C if, besides the correct classification as function or not function, a correct justification was given. In the cases in which the identification was correct, but no justification or inadequate justification was given, the answer was considered to be PC. Moreover, the frequencies of different types of answers to the different items, according to the master's degree and type of representation used, were considered.

Results

In Table 1 we summarize the answers of the pre-service teachers to the different items of the proposed task, according to the three types of answers considered (C, PC, I) and the situation of having no answer (NA). Globally, considering the eight items all together, 16% of the answers were C, 51% were PC, 20% were I and 13% were NA.

Table 1: Frequency of the different types of answers of the pre-service teachers to the 8 items ($n = 37$)

Type of answer	Graphs								Tables				Algebraic expressions			
	a)		b)		d)		g)		c)		e)		f)		h)	
	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2
C	4	1	3	4	5	4	2	–	3	–	9	8	2	2	1	–
PC	16	13	10	3	7	7	12	13	7	9	4	6	11	8	13	12
I	2	–	3	7	4	3	4	1	9	5	6	–	6	4	3	2
NA	1	–	7	–	7	–	5	–	4	–	4	–	4	–	6	–

C-Correct; PC-Partially correct; I-Incorrect; NA-Without any answer.

These results highlighted some difficulties for pre-service teachers in identifying functions. This difficulty is reflected by the fact that the frequency of answers classified as partially correct - due to the absence of justification (21.3%) or inadequate justification (26%) - is higher than for the other types of response (Table 2).

Table 2: Frequency of answers without justification or with an inadequate justification ($n = 37$)

	Graphs								Tables				Algebraic expressions			
	a)		b)		d)		g)		c)		e)		f)		h)	
	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2
No justification	7	7	3	3	1	3	1	3	3	3	3	5	4	3	10	4
Inadequate justification	9	6	7	–	6	4	6	4	4	6	1	1	7	5	3	8

Now, in Table 3, the analysis is deepened by attending to the master's course being taken by the students and to the type of representation. In terms of the master's courses involved in the study, it can be seen that, in all the eight items, the M2 students present a greater number of C, PC and I answers, while M1 students present a greater number of NA.

Table 3. Percentage of the different types of answers in each of the courses and in each type of representation (n=37)

Type of answer	Master course		Type of representation		
	M1	M2	Graphical	Tabular	Algebraic
C	15,8	17,0	15,54	27,03	6,76
PC	43,5	59,8	52,03	35,14	59,46
I	20,0	23,2	18,92	27,03	20,27
NA	20,7	00,0	13,51	10,80	13,51

C-Correct; PC-Partially correct; I-Incorrect; NA-Without any answer.

Considering the type of representation, it can be seen that the number of correct answers in the tabular representation is higher than the ones in graphical and algebraic representations. This arose from the absence of justifications or, in the case of graphical and algebraic representations, inadequate justifications that result in a classification of PC for these answers.

In the next section, some of the students' answers are presented and analysed according to the representation used.

Identifying functions in graphical representations

Among the items where the information is presented through a graph, item a) represents a function and the pre-service teachers justified it saying that: "each object corresponds to one and only one image" (A8, M1; A6, M2). In item g), where the relation represented is also a function, the same justification was given by two pre-service teachers from M1, but by none from M2.

The relations presented in items b) and d) are not functions. For item b), some pre-service teachers stated that it was not a function since "there cannot be two images for each object" (A13, M1) and "to each object corresponds more than one image" (A12, M2). In item d), they said the relation presented is not a function because "to each object corresponds more than one image" (A8, M1); "a unique object has all the images" (A13, M1); "to the object 3 correspond several images" (A1, M2).

Considering the justifications given, we can notice that some pre-service teachers, from both masters' courses enrolled in the study, have a precise notion of function. It is not possible to infer a similar conclusion from PC answers. For items a) and g), some students answered that the relation "is a function", without any justification (A1, M1; A4, M2), while others provided an inadequate justification for their response, as the following statements illustrate: "it is a function because there are two variables involved" (A2, M1; A14, M2); "to each object corresponds an image" (A21, M1; A6, M2).

The same type of answers was obtained for items b) and d). For item b), some students just answered that the example presented "is not a function" (A10, M1; A5, M2), while others considered that it is not a function "since we do know the values of x and y" (A16, M1), or "because to a domain correspond two domains" (A19, M1).

Regarding item d), some students just answered that the example presented "is not a function" (A11, M1; A9, M2), while others added as justification to that answer the statement "because it is a line" (A1, M1; A14, M2), "because there is no correspondence between points of both sets" (A7, M1), or "because it is a line that corresponds to $x = 3$ and in order to be a function there must be two variables" (A15, M2).

Analysing the incorrect answers, in M1 there are students that considered that the example presented in item a) does not represent a function "because there is no correspondence between the points" (A7). For item b), some of the incorrect answers in M1 were due to the idea that the relation presented is a function "because to one coordinate corresponds one and only one object" (A12). Among the incorrect answers by students of M2 to this item, one can find that the relation presented is a function "since there are two variables and it corresponds to a parabola" (A15) or simply "because it is a parabola" (A8).

As far item d) is concerned, the incorrect answers state that the situation represents a function because, for M1 students, "it is a line" (A23) or "to a coordinate corresponds only one object" (A12). The M2 students that incorrectly answered this item also considered that it was a function, because "it is a constant function" (A8).

Last but not least, to item g), the unique incorrect answer that has a justification states that the relation is not a function "since it is a curve" (A11, M1).

Identifying functions in tabular representations

Two of the eight items, items c) and e), present relations using tables. In the case of item c), the relation is a function, since, as A8's answer states, "to each object corresponds one and only one image" (A8, M1), but it was identified as such by only three pre-service teachers. The situation presented in item e) does not represent a function, since "there is more than one y to the same x" (A23, M1); "to the object 1 correspond two images" (A13, M2).

Considering PC answers, in item c), while some students stated only that the relation "is a function" (A7, M1; A15, M2), others justified their answer, writing that "for each x corresponds a y" (A20, M1), or "each object has one image" (A6, M2). Among incorrect answers to this item, there were statements that the relation is not a function because "it just presents a table with some information" (A16, M1), or "every object has more than an image" (A8, M2).

For item e), some of the PC answers just state that "it is not a function" (A19, M1; A7, M2), while in others one can find justifications, such as the answer stating that it is not a function because "you have twice the same value of x" (A11, M1), or "there are two equal objects" (A8, M2).

In the incorrect answers to this item, some students stated that it is a function because "it has two variables x and y" (A2, M1), or because "to a coordinate corresponds only one object" (A12, M1).

Identifying functions in algebraic representations

Of the eight items, two, items, f) and h), present relations using an algebraic representation. For item f), the relation is not a function, and it was identified correctly by only two pre-service teachers from each Master's degree, giving reasons such as "if we imagine its representation on the Cartesian graphic, to the object -2 correspond several images" (A17, M1) or "every image corresponds to a unique object" (A3, M2). In the situation presented in item h), only one pre-service teacher of M1 gave a correct answer, considering that it represents a function because "every object has one and only one image" (A8, M1).

Analysing the PC answers given to item f), we observe that some students state only that the relation "is not a function" (A21, M1; A10, M2), while others justified that conclusion by saying that it is so because "it is a line" (A1, M1), or "it has only one variable" (A14, M2).

Among the incorrect answers to this item, there are justifications for the fact that the relation is a function, including "this value exists in the abscissa x" (A6, M1), or "to the object corresponds an image" (A6, M2).

For h), some PC answers state only that the relation "is a function" (A17, M1; A10, M2), while others present justifications such as "it has two variables" (A5, M1), or "to each value of x corresponds one value of y" (A8, M2).

In the incorrect answers to this item, some students wrote only that "it is not a function" (A19, M1; A7, M2).

Conclusions and implications for mathematics education

The results of this study suggest that most of the pre-service teachers do not have a precise notion of the concept of function. They face difficulties in identifying the essential attributes of a function, strongly evidenced in their written records or in the oral use of the expression 'one and only one', and also in the exploration of different representations, such as tables, graphs and analytical expressions. These results are partially consistent with the ones achieved in other studies and pointing to an incomplete understanding of the concept of function (Steele et al., 2013). However, the difficulties identified in the present study are related not only to the formal definition of the concept, but also to its use.

Usually, in the school context the two essential attributes of the concept of function are not emphasized: 1) it is a binary relation; 2) it is single-valued. Teaching strategies and textbooks tend to state that, in a function, each x element corresponds to one and only one y element. This compressed form of defining a function, in which those two attributes of a function are not explicit, is an obstacle to the understanding of this definition, and contributes to the difficulties faced by the pre-service teachers in justifying the cases where a function is represented.

In general terms, pre-service teachers are more comfortable distinguishing whether a given relationship is or not a function when a tabular representation is presented, than when a graphical representation or even an algebraic representation is involved. This may be due to a greater use of tables, rather than graphs or analytical expressions, in the teaching strategies (especially at early levels). However, it can also be due to the fact that graphical and algebraic representations require more than correspondence between elements. These representations require knowledge about the relationship between ordered pairs and the distinction between variables. It can also happen that the initial learning prevails more over time, but it may simply be that tables are easier for the students to understand.

The algebraic representation seems to be the most difficult one for the pre-service teachers. This is also the representation with the highest number of partially correct answers. This might be due to the difficulty in justifying the answer. In fact, partially correct answers are the most frequent type of answer in all representations.

This analysis about the relation between the representation used and the concept of function is the main contribution brought about by this study, once the approach from most of the studies simply points to some preference for the use of numeric and algebraic representations based on the fact they are the ones used more often (Steele et al., 2013), and fail to actually analyse each representation.

The difficulty that pre-service teachers reveal in connecting the essential attributes of a function across different representations points to a greater use of a single representation when studying the topic of functions, as suggested by Carraher and Schliemann (2007).

In graphical terms, Markovits, Eylon, and Bruckeimer (1998) consider that the students tend to manifest a misconception of linearity that is the idea that the graph of a function is a straight line. This may be related to the predominance in teaching of this type of graph. In this study, such a conjecture is not verified, as evidenced by the similarity between the number of correct answers in the case where the graph is a straight line and in the case where it is a parabola with a horizontal axis of symmetry.

According to Duval (2006), linking the different representations of functions is not easy. Representations are mobilized and developed only if they are transformed into other representations. Thus, highlighting the importance of connections between different representations is central for students' understanding of mathematical concepts. The use of different representations promotes an understanding about what is mathematically relevant in a representation and helps to convert it to another representation and to identify the specific function from the information on that representation.

Acknowledgements

This research was funded by CIEd - Research Centre on Education, project UID/CED/01661/2019, Institute of Education, University of Minho, through national funds of FCT/MCTES-PT.

References

Brown, A., & Mehilos, M. (2010). Using tables to bridge arithmetic and algebra. *Mathematics Teaching in the Middle School*, *15*(9), 532-538.

Carraher, D., & Schliemann, A. (2007). Early algebra and algebraic reasoning. In F. Lester

(Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 669-705). Charlotte, N.C: Information Age Publishing, Inc. and NCTM.

Cuoco, A. (2001). Preface. In A. Cuoco, & F. Curcio (Eds.), *The roles of representation in school mathematics* (pp. ix-xiii). Reston: NCTM.

Dreher, A., Kuntze, S., & Lerman, S. (2016). Why use multiple representations in the mathematics classroom? Views of English and German preservice teachers. *International Journal of Science and Mathematics Education, 14* (Suppl 2), S363-S382.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*, 103-131.

Flores, A. (2002). Geometric representations in the transition from arithmetic to algebra. In F. Hitt (Ed.), *Representations and mathematics visualization* (pp. 9-29). Mexico: North American Chapter of IGPME, Cinvestav-IPN.

Friedlander, A., & Tabach, M. (2001). Promoting multiple representations in algebra. In A. Cuoco, & F. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 173-185). Reston: NCTM.

Goldin, G., & Kaput, J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, & J. Greer (Eds.), *Theories of mathematical learning* (pp. 397-430). Hillside, NJ: Lawrence Erlbaum Associates Inc.

Goos, M., & Bennison, A. (2008). Surveying the technology landscape: Teachers' use of technology in secondary mathematics classrooms. *Mathematics Education Research Journal*, *20*(3), 102-130.

Kleiner, I. (2012). A brief history of the function concept. In I. Kleiner (Ed.), *Excursions in the history of mathematics* (pp. 103-124). New York, NY: Springer, 103-124.

Lesser, L. (2001). Representations of reversal: An exploration of Simpson's paradox. In A. Cuoco, & F. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 129-145). Reston: NCTM.

Markovits, Z., Eylon, B., & Bruckeimer, M. (1998). Difficulties students have with the function concept. In A. Coxford, & A. Shulte (Eds.), *The ideas of algebra: K-12* (pp. 43-60). Reston, VA: National Council of Teachers of Mathematics.

Meira, L. (1998). Making sense of instructional devices: The emergence of transparency in mathematical activity. *Journal for Research in Mathematics Education, 29*(2), 121-142.

Mitchell, R., Charalambous, C., & Hill, H. (2014). Examining the task and knowledge demands needed to teach with representations. *Journal of Mathematics Teacher Education, 17*, 37-60.

Nachlieli, T., & Tabach, M. (2012). Growing mathematical objects in the classroom - the case of function. *International Journal of Educational Research, 51/52*, 10-27.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Ponte, J. (1990). O conceito de função no currículo de matemática [The concept of function in mathematics curriculum]. *Educação e Matemática, 15*, 3-9.

Ponte, J. (1992). The history of concept of function and some educational implications. *The Mathematics Educator*, *3*(2), 3-8.

Rocha, H. (2016). Teacher's representational fluency in a context of technology use. *Teaching Mathematics and its Applications*, *35*(2), 53-64.

Safuanov, I. (2015, February). The history of the concept of a function and its teaching in Russia. *Proceedings of the ninth congress of the European society for research in mathematics education*, 1866-1872.

Steele, M., Hillen, A., & Smith, M. (2013). Developing mathematical knowledge for teaching in a methods course: The case of function. *Journal of Mathematics Teacher Education*, *16*(6), 451-482.

Stein, M., & Bovalino, J. (2001). Manipulatives: One piece of the puzzle. *Mathematics Teaching in the Middle School*, *6*, 356-359.

Tall, D. (1988). Concept image and concept definition. In J. de Lange, & M. Doorman (Eds.), *Senior secondary mathematics education* (pp. 37-41). Utrecht: OW & OC.

Viseu, F., Fernandes, J., & Martins, P. (2017). Exploration of the concept of function in tables and graphs by students of the 3rd cycle. *REIPE*, *E*(1), 11-15.

L. Leite et al. (Eds.) (2019), Proceedings of the ATEE Winter Conference 'Science and mathematics education in the 21st century'
Brussels: ATEE and CIEd (ISBN: 9789082506549)

The concept of a function among prospective teachers

Maria Helena Martinho* & *Floriano Viseu
Centro de Investigação em Educação (CIEd), Universidade do Minho, Portugal
mhm@ie.uminho.pt | fviseu@ie.uminho.pt

Abstract: The study of functions is an important topic in the mathematics curriculum, relevant for the development of abstract thinking, the application of mathematics to everyday situations and the relationship with other domains of knowledge. However, often students do not have a satisfactory understanding of the function. This may be due to several factors, including the different conceptions of function that teachers themselves may have. As trainers of teachers, we investigate which notions of a function prospective teachers hold. The research conducted was qualitative, based on a questionnaire. The relevance of the topic, either from a mathematical or a didactical perspective, entails the need for designing new dynamics in teachers' education that are able to challenge intuitive conceptions and build proper connections.

Keywords: conceptions; function; prospective teachers

Introduction

The complexity of mathematical concept formation makes teacher training courses responsible for promoting identification dynamics and strategies in order to clarify prospective teachers' conceptions about key concepts in the development of mathematical knowledge. Interest in such conceptions is based on the assumption that there is a conceptual substratum that plays a determining role in thought and action (Ponte, 1992), which has relevance for the teaching of a given mathematical concept. Among the various concepts that organize math programmes, the one of a function is particularly notable for "providing a consistent way to make connections between and across a wide range of topics in mathematics itself and in other areas" (Santos & Barbosa, 2016, p. 144). It is a concept of an abstract nature in which many students manifest difficulties in their learning. These difficulties derive, according to Santos and Barbosa (2016), from the "diversity of ways of communicating it and (…) interpreting it, making it a fertile ground for studies of its teaching processes" (p. 114). The relevance of the concept of a function for the development of abstract thinking, and its relationship with other mathematical topics, justifies giving it a central concern in training prospective primary education teachers. In the realization of this objective, we analyse the notion of function in Portuguese school mathematics programmes and discuss the conceptions that prospective teachers hold about its role in mathematical practice.

The notion of a function in the Portuguese mathematics programmes

The Portuguese education system encompasses twelve years before higher education. The first nine years correspond to basic education and the last three to high school. Basic education consists of three cycles, lasting four, two and three years, respectively. The first cycle has a single teacher covering all subject areas. Along these three cycles of basic education, the math's curriculum is the same for all students. In the three years of high school the math's curriculum differs according to the specialization domain the student may take (Science, Humanities, Technology or Arts).

In their curricular organization, the mathematics programmes for the early years include topics that, in informal terms, express relationships somehow connected to the notion of a function. Examples include the result of arithmetic operations between two numbers or the relationship between a given geometrical figure and the measurement of its area. In teaching such topics, the relevance of mathematical communication in stressing language aspects that help students to understand the requirements underlying the notion of a function is revealed. This formally emerges in the 7th grade programme under the topic 'Functions, Sequences and Successions' (MEC, 2013). The topic deals in this grade with the identification of correspondences between sets that translate into functions. Moreover, it introduces the terminology associated with the identification of object, image, domain, target set, and range, as well as independent and dependent variables. This is followed by the representation of functions through arrow diagrams, tables and Cartesian graphs, as well as by a discussion of operations on functions, with the purpose of introducing the notions of linear function ($y = ax, a \neq 0$) and, subsequently, that of direct proportionality.

In the 8th grade, graphs of affine functions ($y = ax + b$) are studied through the identification of corresponding equations in the plane. This aims at making students aware that the graph of an affine function is obtained by translating the graph of a linear function with the same slope, taking into account the meaning of the parameters of the corresponding equation; it also introduces the procedure for obtaining the slope of this line knowing any two points in the domain. Finally, in the 9th grade, algebraic functions are studied through the definition of inverse proportionality, in their different representations (tabular, analytical and graphical), and the graphical interpretation of solutions of 2nd degree equations. This leads to the identification of the curves that represent functions of type $f(x) = ax^2$ (with $a \neq 0$) and the solution set of the 2nd degree equation $ax^2 + bx + c = 0$ (with $a \neq 0$) as the intersection of the equation for a parabola, $y = ax^2$, with the one for a line, $y = -bx - c$.

With the knowledge acquired during their training years at the 3rd cycle, it is intended that students become able to apply it when solving problems in various contexts. Later, in the transition to secondary education, this knowledge serves as a prerequisite for Grade 10, in which the curriculum covers defining the composition of functions and the inverse function of a bijective function, relating geometric properties of graphs to properties of the functions under study, identifying monotony ranges of real-variable real functions, identifying extremes of real-variable real functions, and studying elementary functions and algebraic operations on functions. In the 11th year the study of functions widens to include trigonometric functions, the Heine limits of real functions of a real variable, those derived from real functions of a real variable and their applications. Finally, in the 12th year, limits and continuity of real-variable functions and successions are studied, and the study of derivatives of real functions of a real variable is deepened.

Conceptions about functions

The education of mathematics teachers always assumes a number of epistemological, ideological and cultural positions with respect to teaching, the teacher and the students (Marcelo Garcia, 1999). However, at the level of initial education, the mathematical formation of prospective teachers is characterized by the teacher-knowledge-student relationships, contextualized by the way they observe, as students, the development of the mathematical curriculum (Ponte, 1992). Shulman (1986) points out the importance of didactic research in order to find answers to questions related to teaching content, teachers' knowledge of this content, where and when the latter is acquired, how and why it is transformed during formal training, and, finally, how it should be used in concrete classroom teaching. Cognitive in nature, conceptions are, on the one hand, fundamental in structuring the meaning assigned to reality and, on the other, important as a blocking element for new understandings (Ponte, 1992). The construction of a conception takes place in individuals (as a result of elaboration of their own experience) and at social level (from comparing their own synthesis with those of others). The formulation of mathematical concepts and the teaching/learning of mathematics are thus somewhat influenced by the dominant experiences and social representations. Therefore, the identification of conceptions in prospective teachers, in this case about the notion of function, provides indicators about their mathematical formation.

The applicability of mathematics, as a tool for studying real phenomena, depends on the conception of a given model that synthesizes and relates the main characteristics of the phenomenon to be handled. Such relationships are often represented by functions.

The concept of a function is the result of a long development of mathematical thinking (Caraça, 1984). Such historical evolution refers to the complexity of the construction of the notion of a function, and its acquisition as an abstract concept as a result of several mental constructions developed by mathematicians and scientists to solve problems and create theories (Evangelidou, Spyrou, Elia, & Gagatsis, 2004). This may explain some of the difficulties manifested by many students and prospective teachers in building up their own construction of this concept.

A study by Vinner and Dreyfus (1989) shows that college students during a course on calculus, even if they were able to give a correct explanation of the definition of function, did not apply the definition successfully. Breidenbach et al. (1992) point-out that "college students, even those who have had a good number of math courses, do not have much understanding of the concept of function" (p. 247), confirming that it is a complex concept for students and its conceptual development requires a longer period of time than is typically assigned. Actually, the concept of a function must be dynamically introduced as a kind of relationship, correspondence, or covariation, rather than putting excessive emphasis on the static concept of (a set of) ordered pairs (Hansson, 2004). On the other hand, the diversity of representations associated with functions and the difficulty of establishing connections between them may become confusing on a first exposition. In practice, different teaching approaches to the concept of function arise from what students infer from vague information. Based on this assumption, Evangelidou et al. (2004) conducted a study with prospective teachers, predominantly from primary school pre-service courses, seeking to understand the interpretation of the concept of a function among university students with respect to the conception itself, its use, and the identification of functions in multiple representations. The study revealed three strong trends in prospective teachers' notions about functions. The first is the identification of a function with the more specific concept of 'one-to-one function', common among a large percentage of students. Although this notion works for a wide range of situations involving functions, it becomes a strong obstacle to understanding the concept at a broader level. The second trend is the idea that a function is an analytical relationship between two variables. The third trend simply identifies functions with their representations, e.g. as a diagram or a Cartesian graph.

In the studies by Tall and Vinner, conceptions of the notion of a function are framed by the *concept image*, a mental construction that represents the cognitive structure associated with the concept, which includes all mental images and associated properties and processes, and the *concept definition*, i.e., the formal definition (Tall & Vinner, 1981;

Vinner & Dreyfus, 1989). Despite being introduced to the formal definitions of mathematical concepts, students tend not to use them when asked to identify or construct a concrete, related mathematical object. They often base their ideas on a conceptual image emerging from the set of all mental images associated in the student's mind with the name of the concept, along with all the properties that somehow may characterize them (Vinner & Dreyfus, 1989). Consequently, students' responses to concept-related tasks depend on these conceptions and deviate from expectations of institutional knowledge. Viirman, Attorps, and Tossavainen (2010) identified different categories of definitions and conceptual images concerning the notion of a function:

> *Correspondence/dependence relation.* A function is any match or dependency relationship between two sets that assigns to each element in the first set exactly one element in the other set.
> *Machine.* A function is a 'machine' or one or more operations that transform variables into new variables. In this case, no explicit mention of domain and range is made.
> *Rule/formula.* A function is a rule, a formula, or an algebraic expression.
> *Representation.* The function is identified with one of its representations.

For the authors, students' common images of the concept of a function have direct implications for teaching, as they can be used as a starting point for any prospective teaching of this concept.

Research methodology

In order to investigate the way a function is conceptualised among prospective primary education teachers, the paper focuses on four open questions: (1) What is the meaning given to the term function?; (2) (a) When do you use functions? (b) Who else uses functions and when?; (3) What mathematical symbol(s) do you use to represent functions?; (4) How would you explain the concept of function? The study included 87 prospective teachers who were organized into three groups according to the year they attended, as follows. Group A consisted of 29 students from the 1st year of the bachelor's degree in basic education (S1 to S29); Group B consisted of 40 students from the 3rd year of the bachelor's degree in basic education (S30 to S69); Group C consisted of 18 students from the 2nd year of the master's degree in teaching primary school and Mathematics and Natural Sciences at basic school (S70 to S87). The codification, from S1 to S87, followed the order of the students of each grade considered.

The data was subjected to content analysis and summarized around the following dimensions: meaning, use (when and who), symbolic representation, and concept. The information from data analysis, resulting from the answers given by the different groups of prospective teachers, is presented according to the following categories:

Question 1: Correspondence/ Dependency relationship; Machine; Formula/rule.
Question 2a: School context; Out-of-school context; School and out-of-school context.
Question 2b: Teachers and students; Specific professions; Any professional context.
Question 3: Isolated terminology; Analytical expression; Arrow diagram; Multiple representations.
Question 4: Common language; Algebraic expressions; Arrow diagram; Cartesian graph; Multiple representations.

Results

With respect to the question "What is the meaning given to the term function?", most students mentioned that it is a correspondence or dependency relationship, as shown in Table 1.

Table 1. Frequency of answers from different groups to question 1

Meaning of the term function	Group A	Group B	Group C	Total
Correspondence/ Dependency relationship	24	29	10	63
Machine	1	0	2	3
Formula / rule	0	1	1	2
Meaningless	3	9	5	17
No reply	1	1	0	2
Total	29	40	18	87

Although almost all students refer to the concept of a function as a relationship, it is possible to identify some differences between the responses. Some of them identify "a relationship between two sets" (S9), or stress it "relates to sets, images and objects" (S11), while others refer to a "relationship between variables" (S76). The notion of dependency can also be identified in some of the answers, for example, "function means a transformation of an x by a y. When we have a dependent variable and an independent variable" (S80) or "A function implies the existence of two variables (x, y). By organizing the regular data in a graph, we can predict the values of x or y, knowing other variables" (S55). A very limited number of participants, seven out of the 87, report that in a function, the relationship is single-valued. For example: "It is a function when an element of set A matches one and only one element of set B" (S21) or "A function is a mathematical concept. A function is when one element of the starting set corresponds to one and only one element of the target set" (S52).

Some answers adopt an operational perspective that can be classified as belonging to the category of Viirman et al. (2000): that of a function regarded as a "machine". Examples include "A function is a mathematical method used to find an unknown value" (S23), "a function is something that allows us to determine y corresponding to an x or the opposite in a particular case" (S79), or "function means a transformation of an x by a y. When we have a dependent variable and an independent variable." (S80). Only two students associated the notion of function with a rule or formula, for example: "It consists of an expression with at least 2 unknowns, where one can verify the relationship between them" or "For me, a function is an expression that relates two variables, thus one being a dependent variable of another" (S76).

With respect to the second question, item (a), "When do you use functions?", the answers received were classified in the following categories: School context; Out-of school context; School and out-of-school context; Meaningless (Table 2).

Table 2. Frequency of answers from different groups to question 2.a

When using functions	Group A	Group B	Group C	Total
School context	11	29	11	51
Out-of-school context	9	4	4	17
School and out-of-school context	7	7	2	16
Meaningless	1	0	1	2
No reply	1	0	0	1
Total	29	40	18	87

The most frequently occurring category for each group was the school context. For example: "I only use functions when asked by teachers" (S47), "School or university" (S18), "In mathematics as a way of getting a relationship between two sets" (S3). Some students, on the other hand, argue that functions can be used in any context: "Functions are used when we need to establish relationships between two variables" (S55), "In mathematics classes and sometimes in everyday life" (S27).

For the second question, item b, the students were asked "Who else uses functions, and when?" The answers were organized according to the following categories: Academic context, Specific professions and Any professional context (Table 3).

Table 3. Frequency of answers from different groups to question 2.b

Who uses functions	Group A	Group B	Group C	Total
Academic context	7	13	4	24
Specific professions (engineers, nurses, etc.)	7	12	8	27
Any professional context	9	11	6	26
Meaningless	1	2	0	3
No reply	5	2	0	7
Total	29	40	18	87

Responses, while showing some diversity in terms of who uses functions, are less diverse in what concerns their use. For example, some answers point to the academic context: "Mathematicians and learners" (S1), "teachers in class" (S43), "Mathematicians and students in mathematics, as they are presented with a diverse number of problems where you have to decipher or apply functions" (S53).

Responses emphasizing other professional contexts are more diverse, as one would expect. For example: "Traders and their buyers" (S3), "Architects to carry out projects" (S21), "Who works with money and quantities" (S15), "Taxi drivers when calculating the total value of the fare; any seller who wants to know the full value of a purchase" (S30).

Some participants included in their answers academic and everyday contexts, for example, "Mathematicians mainly, but everybody uses them, on a daily basis, to solve mathematical problems, to calculate unknowns that arise in everyday life" (S31).

In the third question, participants were asked to indicate "What mathematical symbol(s) do you use to represent functions?" (Table 4).

Table 4. Frequency of answers from different groups to question 3

Symbols used to represent functions	Group A	Group B	Group C	Total
Isolated terminology	12	22	5	39
Analytical expression	11	11	8	30
Arrow diagram	2	4	0	6
Multiple representations	0	1	3	4
Meaningless	4	1	2	7
No reply	0	1	0	1
Total	29	40	18	87

Answers classified as "isolated terminology" are for example, "x, y, ( )" (S2); "A lowercase letter, ex. $f(6)$"; "F to represent a function and we have an image and an object" (S24). Several participants, on the other hand, resorted to analytical expressions, such as: "x and $f(x)$, for example, $f(x) = 2x^2$ or $g(x) = 2x + 1$" (S27); "$y = mx + a$; $y = x^2 + a$" (S53).

A small group used the symbolism of arrow diagrams, as, for example, the one shown in Figure 1.

(object) (image)

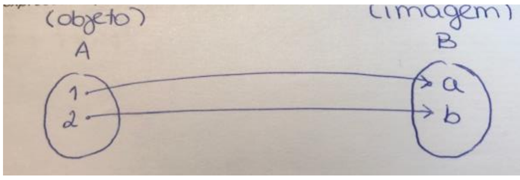


Figure 1. Reply (S22) to question 3

Some students presented more than one representation, as in the example shown in Figure 2. Such answers were classified as “multiple representations”.

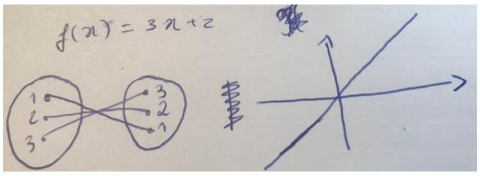


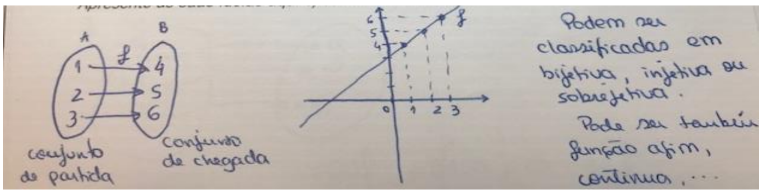
Figure 2. Reply (S80) to question 3

Finally, question 4 asked participants to explain “How would you explain the concept of a function?” The distribution of the answers per category is organized in Table 5.

Table 5. Frequency of answers from different groups to question 4

Explaining of the function concept	Group A	Group B	Group C	Total
Common language	3	3	4	10
Algebraic expressions	1	4	0	5
Arrow diagram	12	18	4	34
Cartesian graph	3	2	4	9
Multiple representations	5	4	5	14
Meaningless	5	9	1	15
Total	29	40	18	87

Some students tried to outline an explanation by resorting to common language, for example: “Something that is associated to something with a particular intent. For example, in 2 weeks I do 2 homeworks, in 4 weeks I do 4. Another example: My brother is 6 years old, I am 9. When he will be 18, I will be 21” (S2). This student provided two concrete examples. Actually, almost all students resorted to concrete examples, although represented in different ways. For example, Figure 3 shows a representation of the concept based on both an arrow diagram and a Cartesian graph. Furthermore, the student S20 adds a reference to some properties associated with the concept of a function: “they can be classified as bijective, injective or surjective”; and also “they can be affine, continuous, …”.



(Starter set) (Target set)

Figure 3. Reply (S20) to question 4

Most students who resort to "multiple representations" apparently seek to capture situations involving both discrete and continuous sets, through the different approaches they have encountered through years of formal education. However, it is not always clear that this distinction is actually assumed by the students. For example, Student S20 depicted a line on the Cartesian graph suggesting a function the domain of which is a continuous set. However, the fact that the elements shown in the Cartesian graph and the arrow diagram are exactly the same may suggest he was just using different representations and that the line is mislabelled. Another example, complementing the one above, is the case of student S44 (Figure 4).

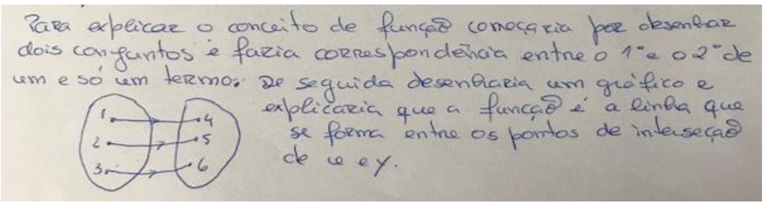

Para explicar o conceito de função começaria por desenhar dois conjuntos e fazia correspondência entre o 1.º e o 2.º de um e só um termo. De seguida desenharia um gráfico e explicaria que a função é a linha que se forma entre os pontos de interseção de x e y.
1 2 3 4 5 6

Figure 4. Reply (S44) to question 4

The student starts from an example presented through an arrow diagram with a discrete set and then states that "To explain the concept of a function, I would start by drawing two sets and match one to the other in terms of one and only one term. Then I would draw a graph and explain that the function is the line connecting the intersection points of x and y" (S44). This answer, like many others, reveals students' weakness in mastering the concept of a function. It should be added that the majority of answers to question 4 are based on examples only. Few students were concerned to provide additionally an explanation.

Conclusions and implications for mathematics education

The study reported in this paper provides a number of hints on how the notion of a function, central as it is in Mathematics, is conceptualised among a sample of students training to become teachers of mathematics. The participants express the concept of a function in terms of a relation between magnitudes, the existence of objects and images, and the construction of graphs, without, however, being precise on the nature of a functional correspondence. No relevant differences were found between the two groups of students under analysis. In general, the content of the answers given by the students attending the MSc degree does not reflect significant differences with respect to the ones given by the students attending the lower degree.

In general, functions are regarded by the participants in this study from a structural perspective, as some sort of set of pairs, rather than as a transformation or a rule, typically expressed as some form of algebraic formula. This is in contrast with findings in similar studies conducted within the same project in The Netherlands and Ireland (Oldham et al., 2019).

Although further comparative research is necessary to clearly identify the prevalence of different patterns in different national contexts, these differences may come from the different approaches to teaching this concept adopted in such contexts. Clearly, the way the study of functions develops throughout the curriculum contributes to shaping prospective intuitions. In Portugal, functions are initially studied in a numerical or tabular representation, to establish a single-valued relation between two sets consisting of few elements. As the first learning is the most resistant to change, this form of teaching may entail too static a perspective on the definition of a function (Hansson, 2004).

Similarly, the extended use of the arrow diagrams detected may be associated with the fact that most of the students considered have taken mathematics up to the 9th grade only, and it is usually through the use of arrow diagrams that the concept is introduced in basic education.

This piece of research also focuses on the way these students consider the applicability of the concept of a function. Actually, participants often stress, albeit in a rather concise way, the applicability of functions to everyday situations, for example to relate two variables in a study or to analyse data. Their technical use, however, may convey the message that the use of functions concerns mainly academics, teachers, engineers and researchers, rather than lay person in the street.

Further systematic research is required however. In any case, it seems urgent to design new dynamics in teachers' education able to put intuitive conceptions in question and build proper connections.

Acknowledgements

This research was funded by CIEd - Research Centre on Education, project UID/CED/01661/2019, Institute of Education, University of Minho, through national funds of FCT/MCTES-PT.

References

Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. *Educational studies in mathematics*, *23*(3), 247-285.

Caraça, B. (1984). *Conceitos fundamentais da Matemática*. Lisboa: Livraria Sá da Costa Editora.

Evangelidou, A., Spyrou, P., Elia, I., & Gagatsis, A. (2004, July). University students' conceptions of function. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 2, 351-358.

Hansson, Ö. (2004). *An unorthodox utilization of concept maps for mathematical statements: Preservice teachers' response to a diagnostic tool*. Research report, Kristianstad University/Luleå University of Technology.

Marcelo Garcia, C. (1999). *Formação de professores - para uma mudança educativa*. Porto: Porto Editora.

Ministério da Educação e Ciência (MEC) (2013). *Programa de matemática para o ensino básico*. Lisboa.

Oldham, E., Martinho, M., Viseu, F., Doggen, R., Price, E., & Leite, L. (2019). *Investigating prospective science and mathematics teachers' meanings for and representations of functions: An international study.* Paper presented at ATEE Annual Conference. Bath: England.

Ponte, J. (1992). Concepções dos professores de matemática e processos de formação. In M. Brown, D. Fernandes, J. Matos, & J. Ponte (Eds.), *Educação matemática: Temas de investigação* (pp. 185-239). Lisboa: Instituto de Inovação Educacional.

Santos, G., & Barbosa, J. (2016). Um modelo teórico de matemática para o ensino do conceito de função a partir de um estudo com professores. *UNIÓN*, *48*, 143-167.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*, 151-169.

Viirman, O., Attorps, I., & Tossavainen, T. (2010). Different views - some Swedish mathematics students´ concept images of the function concept. *Nordic Studies in Mathematics Education*, *15*(4), 5-24.

Vinner, S., & Dreyfus, T. (1989). Images and definitions for the concept of function. *Journal for Research in Mathematics Education*, *20*(4), 266-356.

L. Leite et al. (Eds.) (2019), Proceedings of the ATEE Winter Conference 'Science and mathematics education in the 21st century'
Brussels: ATEE and CIEd (ISBN: 9789082506549)

Botany in Portuguese textbooks: Analysis of seven biology books for high school students

Fernando Santiago dos Santos & Fernando Guimarães
Instituto Federan de Educação, Ciência e Tecnologia de São Paulo, Brasil & Instituto de Educação (CIEC), Universidade do Minho, Portugal
fernandoss@ifsp.edu.br | fernandoguimaraes@ie.uminho.pt

Abstract: Botany has traditionally been considered one of the most neglected areas in Biology, although it is of high importance for the environment, daily life, economy and so forth. This paper analysed how botanical contents are displayed in high-school textbooks used in Portugal for 10th, 11th and 12th grades. The methodological procedures are based on content analysis with *a posteriori* nine categories and sub-categories: i) Figures, ii) Additional information boxes, iii) Language, iv) Concepts, v) Contextualization, vi) Activities, vii) History of Science and Technology, viii) Teacher's guidebook, and ix) Transversal topics. An overview of our analysis has evidenced that the Portuguese textbooks studied lack contextualization and perhaps deeper relations with other areas of knowledge, despite language is used correctly and appropriately in all books, and teachers' guide books are precise and adequate. Textbook analyses are important tools to track the way Science is dealt with by students and teachers.

Keywords: botanical content; secondary level; textbooks

Introduction

Botany has been usually addressed as an uninteresting and boring subject, perhaps the toughest area within the biological sciences, despite of its importance (food, furniture, oxygen production, clothing, transportation, medicine etc.). Such view is intensified with the teaching and learning of excessive scientific names and nomenclature without cultural, social, economic, and historical contexts, as stated by Wandersee and Schlussler (1999).

Studies on aspects of vegetal groups and plants have been reported since immemorial times (Ferri, 1983; Low, Rodd, & Beresford, 1994). Botany is a consolidated area with various specializations, such as anatomy, physiology, ecology, biochemistry and so forth (Mish, 2003). Joly (1987) and some other authors refer to Botany as the *Scientia Amabilis*, although it might be easy to "[...] transform Botany in the most boring blabber talk of names and characteristics, with no connection with the plant world, which is beautiful and diversified, as well as interesting to study" (p. xv-xvi).

Botanical information extracted from nature has given humankind the basis of initial investigations about the surrounding environment, as plants of common occurrence have supplied men with clothes, food, transportation, shelter, medicines, incenses and a myriad of other uses (Thomas, 2010). Up to now, Botany and Zoology have been recognized as the 'central, building blocks' of biological studies, despite many other areas, such as biotechnology, genetics and ecology (Krasilchik & Marandino, 2004).

Several researchers (Cavadas & Guimarães, 2010; Guimarães, 2008; Guimarães & Santos, 2011) have pointed out that Botany teaching at the elementary school level is perhaps one of the most obliterated tasks within all biological contents, as teachers consider it difficult and out of stimuli. Caldeira (2009), Kinoshita, Torres, Tamashiro, and Forni-Martins (2006), and Santos (2006) mention that teachers should have a better and more solid background in Botany so that they could enhance more satisfying educational practices.

In the past fifteen years, we have been investigating how botany is presented in textbooks in Brazil and Portugal by considering the unquestionable use and credibility such materials are usually given. Textbook use is unquestionable and credible, as teachers usually regard them as important tools to teach (Astolfi & Develay, 1990; Baganha, 2010; Bittencourt, 2007; Issitt, 2005). Moreover, they are a social construction, thus embedded with ideologies, and cultural and historical drivers. They convey transpositions of the academic knowledge and, thus, omissions, simplifications, complexifications and errors are commonly found (Chevallard, 1991; Fracalanza & Megid-Neto, 2003; Casper et al., 2014). There has been much research on the importance, use and applications of textbooks, as well as strong criticism regarding their adoption (Bizzo, 2000; Massabni & Arruda, 2010). The Education Bureau (2016) states that textbooks play an important role in supporting learning and teaching in schools, and they ought to enable students to seek various ways of learning; thus, learners might have opportunities, in accordance to their own interests and needs, to enhance various skills of learning.

Guimarães (2019) has recently published a study investigating how Natural Sciences in Portugal are dealt with in primary school textbooks; this study is an extensive analysis of textbooks in the past hundred years. Regarding textbook selection and evaluation, Souza and Dionisio (2011) carried out an interesting study of how teachers assess didactic materials in Portugal, pointing out that the two most important features focused by teachers is quality and adequateness to students' ages.

As far as botany teaching is concerned, Hershey (1996), one of the first authors to deal with such topic, and more recently Uno (2009), have stressed the need for a more comprehensive, contextualized and effective methodology so that teachers can teach botanical contents and attract their students' attention.

The secondary school level (High School) in Portugal comprises a three-year period (10^{th}, 11^{th}, and 12^{th}); the curricular component 'Biology and Geology' is offered in the first two years, being part of the syllabus "Specific Formation" (there is another, "General Formation", with several other subjects, such as French, Physical Education, Portuguese and so forth). 'Biology' is offered in the 12^{th} year (Decree-Law 55/2018, of July 6).

This paper presents the results from the first author's post doctorate program at the University of Minho (Braga, Portugal). There is no report on the analysis of secondary level textbooks in Portugal concerning Botany; thus, our results are unprecedented.

Our research focus relies on: a) To analyse how botanical contents are displayed in high-school textbooks used in Portugal for 10^{th} and 11^{th} (Biology-Geology) and 12^{th} (Biology) grades; b) To characterize the material analysed in terms of compliance with the Portuguese Guidance for the high-school curriculum.

Research methods

The methodological procedures are based on content analysis as proposed by Bardin (1994), and modified by Fonseca Jr. (2006), having in mind that such methodological approach is a useful tool to assess textbooks.

Content analysis is a set of techniques aiming at analysing communications and describing contents with systematic procedures. It supplies researchers with guidelines that allow inferences of knowledge regarding conditions of production and reception of such messages. This technique also deals with various forms of communication amongst men so that subjacent messages in a text are found. It is particularly suitable for printed materials, as it may be used as many times as needed; thus, for textbooks, it is a common practice. As Bardin states, "[content analysis] is the manipulation of messages [i.e., content and expression of such content] to evidence guidelines that might allow us to infer upon other realities other than the message itself [...]' (Bardin, 1994, p. 46).

Content analysis goes through three steps (or phases): a) previous analysis (during which researchers make a list of text features); b) inference (includes logical deduction, where we may emphasize causes and consequences relative to the first descriptions of messages); and, c) interpretation (the true meaning of described messages).

During the previous analysis, Fonseca Jr. (2006) mentions the importance of the categorization process, which is a method of selecting, counting, and classifying representative units of text into categories (and sub-categories). Bardin (1994, p. 117) says that "categorization is a classifying operation of the building elements out of a group or set of things, through differentiation, and, subsequently, through re-grouping with genre (analogies) by considering previously defined criteria".

In our study, nine *a posteriori* categories and sub-categories were created to analyse botanical contents (Table 1). Each of the categories and their sub-categories are explained and commented in detail in the section 'Results and discussion'.

Table 1. Categories and sub-categories created for the textbook analysis

Category	Description	Sub-categories
Figures	Inclusion of any graphic material which might enhance students' perception of structures, physical appearance etc.	Pictures (photographs)
		Size scale
		Illustrations
		Graphs
Boxes	Side or bottom square or equivalent with information which is not present in the main text	Occurrence
		Curiosity
Language	Use of the Portuguese language in accordance with the New Orthography	Correct use
		Adequateness to age
		Glossary
Concepts	Terms of essential understanding of ideas, theories and alike	Updating and precision
		Conceptual problems
Contextualization	Insertion of botanical contents into culture, economy and so forth	Occurrence
		Interdisciplinary approach
Activities	Exercises aimed at revising and/or enhancing further studies	Type
		Field/laboratory practice
History of Science and Technology	Naturalists/scientists and/or scientific episodes/events related to Botany	Occurrence
		Type of citation
Teacher's guidebook	Specific material designed for teachers	Didactic approach
		Assertive and direct language
Transversal topics	Occurrence of other areas of knowledge which are not normally related to Botany	Occurrence
		Relevance to students

Content analysis was applied to seven high-school textbooks, chosen in accordance with the nationwide textbook approval carried out by the Portuguese Ministry of Education (DGE, 2018), and effectively used in secondary level schools in Portugal (Table 2). Each material was scrutinized in details, looking for any botanical content, as part of the previous analysis.

Table 2. Textbooks used for the present work (YP = year of publication)

Authors	**Title/Editorial company**	**YP**
Amparo Dias da Silva et al.	Terra, Universo de Vida (10th year)/Porto Editora	2012
	Terra, Universo de Vida (12th year)/Porto Editora	2016
Osorio Matias and Pedro Martins	Biologia 10/Areal Editores	2018
	Biologia 11/ Areal Editores	
	Biologia 12/Areal Editores	
Joao Carlos Silva et al.	BioDesafios 12/ASA Editores	2011
	BioDesafios 12º ano/ASA Editores	2014

Results and discussion

During the inference and interpretation steps of content analysis, we found that botanical contents were found in all of the materials analysed, in all grades (10th to 12th). In Table 3, an explanation of the categories of analysis is presented, and Table 4 shows how information extracted from our analysis was condensed for a broader view of the material selected.

Table 3. Explanation of the categories of analysis

Dimensions of analysis	**Acronym**	**Sub-dimensions**
Figures	Fig	Q = quantity P = photograph/picture S = size scale IG = illustrations and/or graphs
Additional information boxes	Box	O = occurrence C = curiosity
Language	Lang	U = correct use of Portuguese A = adequateness to student's age G = glossary or similar tool
Concepts	Conc	U = updating and precision C = conceptual problems
Contextualization	Cont	O = occurrence I = interdisciplinary relations
Activities	Activ	O = occurrence Q = questionnaire-like F = field/laboratory practice
History of Science and Technology	HCT	O = occurrence T = type B = information in a box T = information in the main text E = historical context of the evolution of a certain scientific concept
Teacher's guidebook	TG	D = didactic approach A = direct and assertive language
Transversal topics	Trans	O = occurrence R = relevance to students
In all cells: Y = yes/occurrence; Pr = partially present; N = no/absence.		

Table 4. Condensed information regarding the content analysis carried out with the seven textbooks (numbered in the first column, 1 through 7)

Txt #	CATEGORIES																					
	Fig				Box		Lang			Conc		Cont		Activ			HCT		TG		Trans	
	Q	P	S	IG	O	C	U	A	G	U	P	O	I	O	Q	C	O	T	D	A	O	R
1	39	Y	N	Y	N	N	Y	Pr	Y	Pr	Y	N	N	Y	Pr	Y	N	N	Y	Y	Pr	Pr
2	49	Y	N	Y	N	N	Y	Y	N	Y	Y	Pr	N	Y	Pr	Pr	Y	B/E	Y	Y	N	N
3	19	Pr	N	Y	N	N	Y	Pr	Y	Y	N	Y	Y	Y	Y	Pr	Y	T/E	Y	Y	Y	Y
4	35	Pr	N	Y	Y	Y	Y	Pr	N	Y	N	Y	Y	Y	Y	Y	Pr	T/E	N	N	Y	Y
5	42	Pr	N	Y	N	N	Y	Pr	N	Pr	Y	N	N	Y	Pr	N	N	N	N	N	N	N
6	47	Y	N	Y	N	N	Y	Y	N	Y	N	Y	Y	N	N	N	Pr	T/E	N	N	Y	Y
7	4	N	N	Y	N	N	Y	Pr	N	Y	N	Y	Y	Y	Pr	Y	N	N	N	N	Y	Y

Category 'figures'

Figures are not shown with size references or scales in any material (0%), although all materials (100%) present figures, mostly photographs (Figure 1).

Illustrations and graphs are shown very occasionally: the first ones are present especially when complex mechanisms or phenomena (e.g., photosynthesis) would require a series of elaborate (and perhaps expensive) sets of photographs; graphs are evident mainly when presenting statistics and related subjects (e.g., number of seeds of a certain plant produced during a season).

The existence of pictorial material in a textbook is an important didactic trace, as students seem to be more interested in a certain subject when pictures and, mainly, photographs add colour, shape and three-dimensional perspectives (Santos, Costa, & Santos, 2018; Vidal Junior & Kock, 2013).

The lack of size references or scales, on the other hand, might be an issue as students (and teachers) do not grasp the real magnitude of living beings and their structures; for instance, a sequoia (which may not be found in the neighbourhood) may be compared to a common pine tree, or a chloroplast may have no direct connection in the student's perception of microscopic bodies.

Category 'boxes'

Additional information boxes with curiosities were found in only one material (14.3%). These boxes usually provide students and teachers with additional data which are not present in the main text (Pacheco, 1996).

Although suggestions on additional material may be present in the teacher's guidebook, textbooks with boxes might drive students into complimentary studies by reading such information. Moreover, bridges to other areas of knowledge may also be enhanced when boxes bring information with which teachers and students have the chance to work on other curricular topics: in Figure 2, for example, the numbers may be used to statistical and mathematical problems and situations. Note that scales/size references are not present.

Figure 1. Examples of photographs shown in one of the textbooks

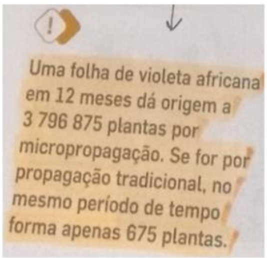
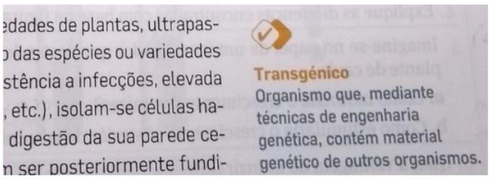

Uma folha de violeta africana em 12 meses dá origem a 3 796 875 plantas por micropropagação. Se for por propagação tradicional, no mesmo período de tempo forma apenas 675 plantas.

Figure 2. Example of a lateral box found in one of the books regarding vegetative propagation

Category 'language'

All materials use Portuguese appropriately (100%), but glossaries or term explanations were found in only two materials (28.6%). We noted that the Portuguese language seemed to be use appropriately towards students' ages at such schooling period in two materials (28.6%); in the remaining materials (71.4%), language was partially adequate, as it sometimes involves difficult concepts and even grammatical constructions, being appropriate, perhaps, to undergraduate students.

Caldeira (2009) discussed that biological and/or scientific terms are usually difficult to be grasped, especially when they are too specific or abstract (such as molecular, genetic and biochemical structures); thus, the presence of glossaries might ease the understanding of such peculiar terms. Figure 3 brings an example of a concept defined in a side box of one of the textbooks.


edades de plantas, ultrapas-
o das espécies ou variedades
stência a infecções, elevada
, etc.), isolam-se células ha-
digestão da sua parede ce-
n ser posteriormente fundi-

Transgénico
Organismo que, mediante técnicas de engenharia genética, contém material genético de outros organismos.

Figure 3. An example of a term defined in a glossary: definition of a transgenic organism

Category 'concepts'

Conceptual problems occurred in four materials (57.1%), mostly related to simplifications and omissions. An example of a simplification for a High School textbook is the extreme reduction of the photosynthetic process into a simple $6H_2O + 6CO_2 =$

$C_6H_{12}O_6 + 6O_2$; an omission is, for instance, the suppression of information (as when one says that green algae are responsible for most of the ocean-producing oxygen that goes into the atmosphere - many other groups of microscopic, photosynthetic organisms also produce oxygen and release it into the atmosphere, such as cyanobacteria). As seen in Figure 4, books still present obsolete information; updating information may be an issue for textbook editors, as new discoveries and taxonomic placement (just to mention few examples) sometimes take years or decades to be added to more recent editions.

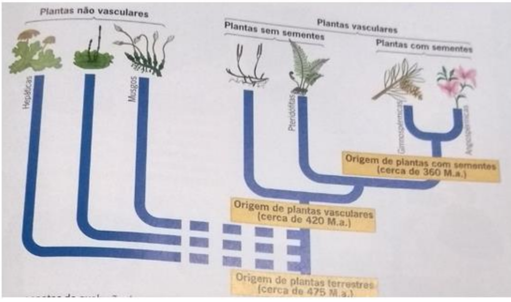


Figure 4. Phylogenetic cladogram

The above cladogram brings phylogenetic issues, such as the inappropriate terms 'pteridophyte' and 'gymnosperms', which are no longer valid (these are polyphyletic clades); moreover, the picture does not show the name of the second group (left to right, which is a hornwort, or Anthocerotophyta).

Category 'contextualization'

Botanical contents were contextualized in five materials (71.4%), although very reduced or almost imperceptible in one material (14.3%).

Bostick, van Dyke, and Stucky (2000) stated that by supplying students with contexts (economical, financial, ecological, social and so forth), botany teaching might be enhanced greatly because students tend to get more interested in a subject when it has commitments to their lives. Our world has changed rapidly, and menaces to our existence are the focus of international debates, such as global warming, famine, transgenic organisms, organic agriculture, just to name a few issues.

Figueiredo, Amaral, and Coutinho (2012) point out that interdisciplinary approaches should be encouraged, especially those that are pertinent to routine life; Figure 5 brings an example of how botanical contents might help students understanding his surrounding environment by visualizing its context.

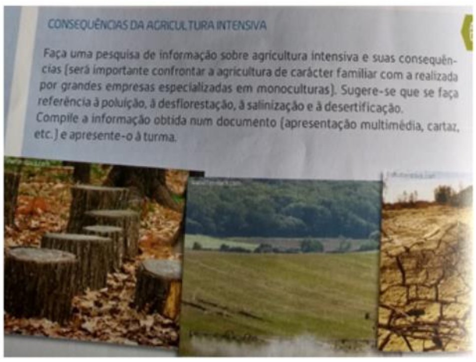
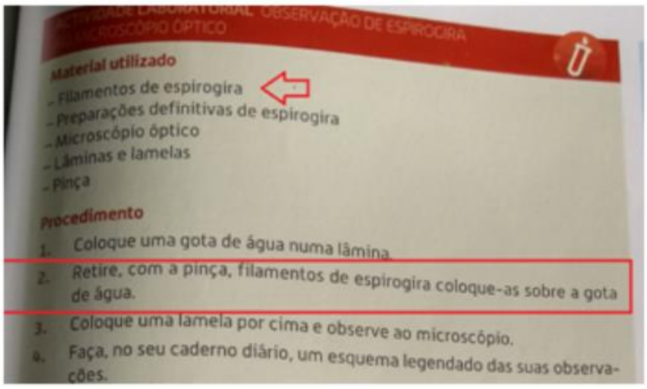


Figure 5. Example of contextualization: consequences of intensive agricultural practices

Category ‘activities’

Activities were found in all but one material (85.7%), generally a mixture of questionnaire-like activities and laboratory/field work tasks. Moreno, Reis, and Calefi (2016) discussed that students should be exposed to a diversified set of activities (home research, project development, data collection etc.), and not only (or almost exclusively) to questionnaire-like questions, which do not demand much of the student’s curiosity and problem-solving skills, abilities to be enhanced and reinforced (UNESCO, 2003).

Even laboratory or field work may present some issues, as materials may not be easily found or reproduced: spirogyra filaments are not frequently found in freshwater bodies (Figure 6).


OBSERVAÇÃO DE ESPIROGIRA

Material utilizado
- Filamentos de espirogira
- Preparações definitivas de espirogira
- Microscópio óptico
- Lâminas e lamelas
- Pinça

Procedimento
1. Coloque uma gota de água numa lâmina.
2. Retire, com a pinça, filamentos de espirogira coloque-as sobre a gota de água.
3. Coloque uma lamela por cima e observe ao microscópio.
4. Faça, no seu caderno diário, um esquema legendado das suas observações.

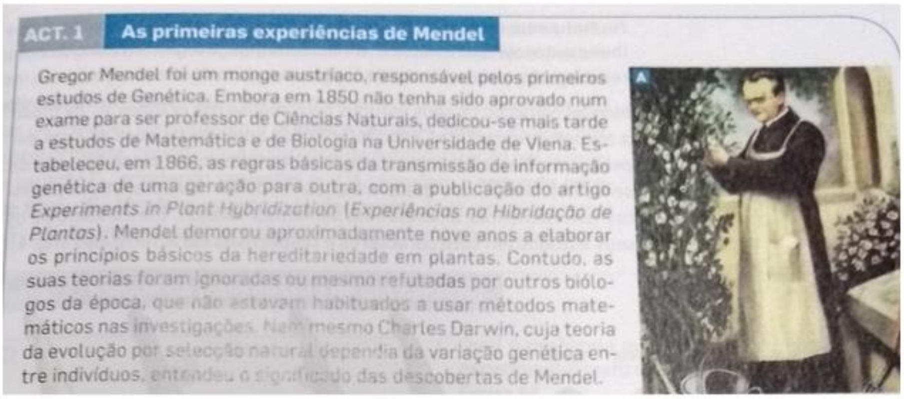
Figure 6. Excerpt of an activity aimed at visualising green algae (*Spirogyra*)

Category ‘history of science and technology’

Botany in a historical approach is basically based on biographies of scientists and found in only two materials (28.6%); most of this information is presented in the main text, and in only one material (14.3%) it was shown in a side box.

When science and technology are presented in a linear and superficial way, students tend to misunderstand how scientific thought is actually built, thus leading students to have a shallow perspective of scientific endeavours (Vidal & Porto, 2012).

It is important to note that out of the seven books analysed, only two books mentioned something related to Botany in a historical perspective, even though in problematic ways (Figure 7). Note that his dates of birth and death are not shown. Also, he did not establish the basic rules for genetic information transmission, as written in the text.

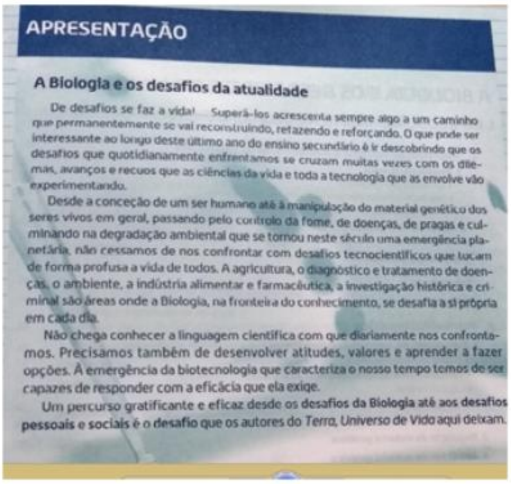

ACT. 1 As primeiras experiências de Mendel

Gregor Mendel foi um monge austríaco, responsável pelos primeiros estudos de Genética. Embora em 1850 não tenha sido aprovado num exame para ser professor de Ciências Naturais, dedicou-se mais tarde a estudos de Matemática e de Biologia na Universidade de Viena. Estabeleceu, em 1866, as regras básicas da transmissão de informação genética de uma geração para outra, com a publicação do artigo *Experiments in Plant Hybridization* (*Experiências na Hibridação de Plantas*). Mendel demorou aproximadamente nove anos a elaborar os princípios básicos da hereditariedade em plantas. Contudo, as suas teorias foram ignoradas ou mesmo refutadas por outros biólogos da época, que não estavam habituados a usar métodos matemáticos nas investigações. Nem mesmo Charles Darwin, cuja teoria da evolução por selecção natural dependia da variação genética entre indivíduos, entendeu o significado das descobertas de Mendel.

Figure 7. An example of a text concerning Gregor Mendel and his experiments with peas in the XIX century

Category 'teacher's guidebook'

Teacher's guidebooks are precise and appropriate in three materials (42.9%%), with didactic approaches and direct, assertive language. These seem to bring relevant, objective and coherent information regarding the material to be used with students (Figure 8).



APRESENTAÇÃO

A Biologia e os desafios da atualidade

De desafios se faz a vida! Superá-los acrescenta sempre algo a um caminho que permanentemente se vai reconstruindo, refazendo e reforçando. O que pode ser interessante ao longo deste último ano do ensino secundário é ir descobrindo que os desafios que quotidianamente enfrentamos se cruzam muitas vezes com os dilemas, avanços e recuos que as ciências da vida e toda a tecnologia que as envolve vão experimentando.

Desde a conceção de um ser humano até à manipulação do material genético dos seres vivos em geral, passando pelo controlo da fome, de doenças, de pragas e culminando na degradação ambiental que se tornou neste século uma emergência planetária, não cessamos de nos confrontar com desafios tecnocientíficos que tocam de forma profusa a vida de todos. A agricultura, o diagnóstico e tratamento de doenças, o ambiente, a indústria alimentar e farmacêutica, a investigação histórica e criminal são áreas onde a Biologia, na fronteira do conhecimento, se desafia a si própria em cada dia.

Não chega conhecer a linguagem científica com que diariamente nos confrontamos. Precisamos também de desenvolver atitudes, valores e aprender a fazer opções. À emergência da biotecnologia que caracteriza o nosso tempo temos de ser capazes de responder com a eficácia que ela exige.

Um percurso gratificante e eficaz desde os **desafios da Biologia até aos desafios pessoais e sociais** é o **desafio** que os autores do *Terra, Universo de Vida* aqui deixam.

Figure 8. An example of a page introducing the book to the teacher

In the case of the materials commented above, there is particular care in aligning pedagogical recommendations and directions with those suggested by the Portuguese Ministry of Education, which recommend that Science/Biology teachers should consider that "[...] the purposes of the Biology education ought to be addressed towards the scientific education of all citizens [...]" (Dispatch 8476-A/2018, August 31); moreover, the same document says:

> "[...] it is important that young people are prepared to face [...] both the scientific and the technological issues brought by societies [...] the study of conceptual, procedural and behavioural contents in Biology makes it possible to comprehend working methodologies used by specialists, analyses of crucial moments in the history of Biology and, also, the instrumental value of scientific and technological skills to grasp problems that may affect people's lives" (Dispatch 8476-A/2018, August 31).

The teacher's guidebooks analysed bring information to teachers that cover basically the above recommendations.

Category 'transversal topics'

Transversal topics are generally centred in ecology, economy and agriculture, but also in medicine (Figure 9). They occur in five materials (71.4%).

Amorim (1998), and Güllich and Araujo (2003) suggested that textbooks contextualize subjects in a transversal way, so that students perceive the importance of science and, particularly, Botany in their own lives. Such trend is also shared by Auler, Dalmolin, and Fenalti (2009).

The books analysed, in which it was possible to note the presence of transversal topics, brought relevant information, thus trying to make students ponder on their own lives and the importance of botanical contexts and contents in such realities.

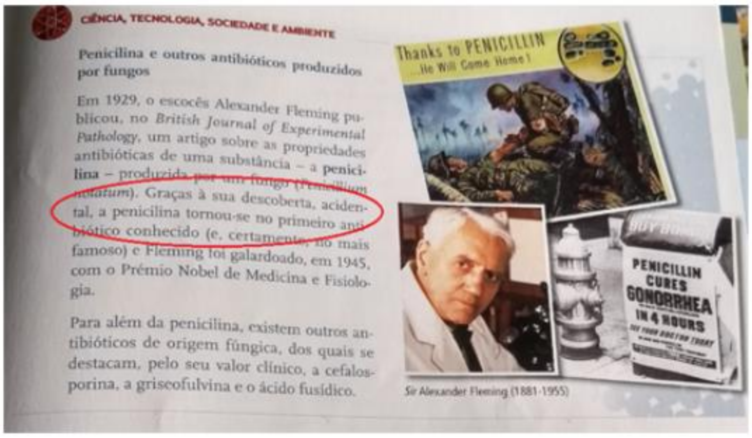


Figure 9. Example of how science positively interferes in our lives: in this case, the development of penicillin from Fleming's studies with bacteria

Final words

By considering the importance of textbooks during schooling (despite some critiques otherwise), and analysing them with our categories, we realized that the seven books considered in this paper are partially appropriate for teaching and learning Botany. Our assumption is based upon two statements: a) "[...] in our uncertainty regarding the future, where there is a myriad of new opportunities for the human development, it is necessary to have students develop competencies that might allow them to question established knowledge by integrating emerging skills, communicating effectively, and solving complex problems" (Decree-Law 55/2018, of July 6, p. 2928-2929); b) every teacher should be aware that textbooks are meant to present, in a didactically structured way, a certain knowledge that students should grasp, understand and make it somehow valid for his own life - the best book will depend upon the material conditions in which both teachers and students are immersed (Baganha, 2010).

Our nine *a posteriori* categories might not be fully sufficient for a straightforward observation that these textbooks may supply users with whole and appropriate materials.

Figures are not shown with size references or scales in any material analysed; this fact may lead students to misunderstand the real size of beings and structures. By considering that living beings come in microscopic and gigantic sizes, it is important that scales or size indicators are presented in textbooks. Only one material brought such resource.

The boxes with additional data may bring students (and teachers) information regarding curiosities, complementation of texts, suggestions of home research and so forth. Although they are not mandatory items in a textbook, additional boxes are useful tools and should be present.

Major mistakes were not found in any of the books analysed, but glossaries of new concepts and/or more complex terms were very scarce. It is noteworthy to mention that botany is a very complex area and its nomenclature may be difficult to gasp, so the occurrence of glossaries in such few materials is something to ponder.

Conceptual mistakes were not common in the textbooks analysed; most issues found concerned simplifications and omissions. There should be no such problems in textbooks, but usually authors and/or publishers do not have appropriate updating of information and, therefore, commitment to avoid spreading over-simplified data.

The occurrence of botanical contents in only half of the materials shows us that textbooks should be more carefully devised, as contextualized topics might enhance curiosity, better understanding, and more meaningful relations with daily life.

There should be a variety of activities (home research, experimentation, projects, field work, laboratory practice etc.) so that students may grasp botanical skills more appropriately and effectively. The activities of the analysed textbooks were questionnaire-like exercises mixed with scarce field and/or laboratory work (generally, experiments with known results, lacking creativity and more active practice).

Only two materials presented some texts with a historical background when dealing with botanical aspects. Even so, biographies of scientists were the most common approach. In general, textbooks lack a broad view of how concepts and ideas were historically built, as well as how technology has taken place in practically all aspects of our lives.

Most materials present botany in a transversal way, usually associated with agriculture, ecology and economy. Although these are important areas of our lives, there should be more transversal topics such as medicine (plants as medicinal drugs), biotechnology, arts and human welfare.

Regarding the general guidelines of the Portuguese Ministry of Education, which consider Biology and Geology as " [...] crucial scientific areas for the development of responsible citizenship" (Dispatch 8476-A/2018, August 31), we see these textbooks as tools that may help students comprehend problems and make decisions upon questions that affect both societies and our earthly subsystems. Yet, we cannot see these materials as enhancers of democratic citizenship (as the Portuguese Government suggests), but as a way of realizing how Science - and, particularly, Botany - is brought to perspective in a society which is gradually welcoming new technologies and skills for the demands of the XXI century.

References

Amorim, A. (1998, May). Interações entre biologia/tecnologia/sociedade no ensino médio: perspectivas para a unidade entre teoria e prática no currículo. *ENDIPE: Encontro Nacional de Didática e Prática de Ensino. Anais e Resumos*. Águas de Lindoia, SP.

Astolfi, J., & Develay, M. (1990). *A didática das ciências*. Campinas, Brazil: Papirus.

Auler, D., Dalmolin, A., & Fenalti, V. (2009). Abordagem temática: natureza dos temas em Freire e no enfoque CTS. *Alexandria Revista de Educação em Ciência e Tecnologia, 2*(1), 67-84.

Baganha, D. (2010). *O papel e o uso do livro didático de ciências nos anos finais do ensino fundamental* (Masters dissertation). Retrieved from https://acervodigital.ufpr.br/bitstream/handle/1884/26239/IMPRESSAO_biblioteca_UFPR.pdf?sequence=1&isAllowed=y

Bardin, L. (1994). *Análise de conteúdo*. Lisboa: Edições 70.

Bittencourt, C. (Org.). (2007). Livro didático: Educação e história. *Simpósio Internacional do Livro Didático. Anais e Resumos*. São Paulo: FAPESP/CAPES.

Bizzo, N. (2000). A avaliação oficial de materiais didáticos de Ciências para o ensino fundamental no Brasil. *VII Encontro "Perspectivas do Ensino de Biologia" - Simpósio Latinoamericano do IOSTE, coletânea,* 23-27. São Paulo: FEUSP.

Bostick, G., van Dyke, G., & Stucky. J. (2000). Enhancing a botany learning environment. *NACTA Journal*, *44*(4), 12-16.

Caldeira, A. (Org.). (2009). *Ensino de ciências e matemática, II: Temas sobre a formação de conceitos*. São Paulo: Cultura Acadêmica.

Casper, S. (2014). Textbooks today and tomorrow: A conversation about history, pedagogy, and economics. *Journal of American History*, *100*(4), 1139-1169.

Cavadas, B., & Guimarães, F. (2010). As ilustrações dos manuais de botânica de Seomara da Costa Primo. In J. Duarte (Org.), *Manuais escolares e dinâmica da aprendizagem: Podem os manuais contribuir para a transformação da escola?* (pp. 1-13). Lisboa: Edições Universitárias Lusófonas.

Chevallard, Y. (1991). *La transpostion didactique: Du savoir savant au savoir enseigné*. Grenoble: La Pensée Sauvage Éditions.

DGE (2018). *Manuais escolares*. Retrieved from http://www.dge.mec.pt/manuais-escolares

Education Bureau (2016). *Textbook Committee*. Hong Kong: Government of the Hong Kong Special Administrative Region.

Ferri, M. (1983). *Botânica: morfologia externa das plantas (organografia).* São Paulo: Nobel.

Figueiredo, J., Amaral, F., & Coutinho, F. (2012). O ensino de botânica em uma abordagem ciência, tecnologia e sociedade. *II Seminário Hispano-brasileiro de avaliação das atividades relacionadas com Ciência, Tecnologia e Sociedade. Anais e Resumos*. Anália Franco, São Paulo.

Fonseca Jr., W. (2006). Análise de conteúdo. In J. Duarte, & A. Barros (Orgs), *Métodos e técnicas de pesquisa em comunicação* (pp. 280-315). São Paulo: Atlas.

Fracalanza, H., & Megid-Neto, J. (2003). O livro didático de ciências: O que nos dizem os professores, as pesquisas acadêmicas e os documentos oficiais. *Contestado e Educação*, *2*, 24-35.

Guimarães, F. (2008). Saberes escolares de botânica nos livros didácticos de Ciências da Natureza dos ensinos primário e básico (1.º ciclo): Análise ao seu estatuto curricular do último século em Portugal. *Plures - Humanidades*, *Ribeirão Preto - SP*, *10*, 27-45.

Guimarães, F., & Santos, F. (2011). A botânica escolar nos ensinos primário e básico (1.º ciclo) no último século em Portugal: Análise de manuais escolares de Ciências da Natureza. *Revista de Educação*, *XVIII*, 83-111.

Guimarães, F. (2019). Research on primary school Natural Sciences textbooks in Portugal. In J. Rodriguez, T. Garcia, and E. Bruillard (Eds), *IARTEM 1991-2016: 25 years developing textbook and educational media research* (pp. 165-174). Santiago de Compostela, Spain: Andavira Editores S. L.

Güllich, R., & Araújo, M. (2003). *A botânica e seu ensino: História, concepções e currículo.* Itajaí, SC: Edições Ijuí.

Hershey, D. (1996). A historical perspective on problems in botany teaching. *The American Biology Teacher*, *58*(6), 340-347.

Issitt, J. (2005). Reflections on the study of textbooks. *History of Education*, *33*(6), 110-121.

Joly, A. (1987). *Botânica: Introdução à taxonomia vegetal*. São Paulo: Editora Nacional.

Kinoshita, L., Torres, R., Tamashiro, J., & Forni-Martins, E. (Orgs.) (2006). *A botânica no ensino médio: Relatos de uma experiência transformadora.* São Carlos, SP: Rima.

Krasilchik, M., & Marandino, M. (2004). *Ensino de ciências e cidadania*. São Paulo: Moderna.

Low, T., Rodd, T., & Beresford, R. (1994). *Magic and medicine of plants*. Australia: Pty Limited.

Massabni, V., & Arruda, M. (2000). Considerações sobre o conteúdo do livro didático de Biologia. *VII Encontro "Perspectivas do Ensino de Biologia" - Simpósio Latinoamericano do IOSTE. Coletânea*, 43-46. São Paulo: FEUSP.

Mish, F. (Org.). (2003). *Merriam-Webster's collegiate dictionary*. Springfield, MA (USA): Merriam-Webster Inc.

Moreno, M., Reis, M., & Calefi, P. (2016). Concepções de professores de biologia, física e química sobre a aprendizagem baseada em problemas (ABP). *Revista Hipótese*, Itapetininga, *2*(1), 104-117.

Pacheco, D. (1996). Um problema no ensino de ciências: Organização conceitual do conteúdo ou estudo dos fenômenos. *Educação e Filosofia, 10*(19), 63-81.

Santos, F. (2006). A botânica no ensino médio: Será que é preciso apenas memorizar nomes de plantas? In C. Silva (Org.), *Estudos de história e Filosofia das Ciências: subsídios para aplicação no ensino* (pp. 223-244). São Paulo: Editoria da Física.

Santos, A., Costa, P., & Santos, F. (2018). *Glossário ilustrado de botânica: Subsídio para aplicação no ensino*. São Paulo: Edições Hipótese.

Sousa, J. & Dionisio, M. (2011). Evaluation and selection of textbooks in Portugal: Perceptions from mother tongue teachers. *The Eleventh International Conference on Research on Textbooks and Educational Media*, 104-112.

Thomas, K. (2010). *O homem e o mundo natural: Mudanças de atitudes em relação às plantas e aos animais (1500-1800).* São Paulo: Companhia das Letras.

UNESCO (2003). *A ciência para o século XXI: Uma nova visão e uma base de ação.* Brasília, Brasil: ABIPTI.

Uno, G. (2009). Botanical literacy: What and how should students learn about plants?. *American Journal of Botany*, *96*(10), 1753-1759.

Vidal, P., & Porto, P. (2012). The History of Science in the PNLEM 2007' Chemistry Textbooks. *Ciência & Educação*, *18*(2), 291-308.

Vidal Júnior, J., & Koch, I. (2013, November). O ensino de botânica no Brasil: Revisão histórica e discussão a partir da análise de livros didáticos e das reformas curriculares. *64ª Congresso Nacional de Botânica. Anais e Resumos*. Retrieved from https://www.botanica.org.br/trabalhos-cientificos/64CNBot/resumo-ins17988-id6316.pdf

Wandersee, J., & Schlussler, E. (1999). Preventing plant blindness. *American Biology Teacher*, *61*, 84-86.

Genetically modified and transgenic organisms: A study focusing on 9th grade Portuguese textbooks

Luísa Carvalho & Luís Dourado
Centro de Investigação em Educação (CIEd), Universidade do Minho, Portugal
luisamscarvalho@gmail.com | ldourado@ie.uminho.pt

Abstract: The production and use of genetically modified organisms (GMO) and transgenic organisms (TO) are among the most controversial socio-scientific issues of our times. Recent research studies suggest that the theme is hardly approached in schools, that students hold a variety of alternative conceptions on those issues and that school textbooks which deal with them do not do it in the best way. Besides, there is some empirical indication that the teaching approach used to deal with the issues that are at stake may make a difference with regard to overcoming students' alternative conceptions. Therefore, this study aims at finding out how six 9th grade Natural Sciences textbooks that are in use in Portuguese schools approach the GMO and TO issues, focusing on three overarching dimensions of analysis: conceptual, methodological and graphical. Results indicate that textbooks do not present and relate adequately the concept of GMO with TO and tend to focus more in advantages than the risks of its production and use. In addition, textbooks offer mainly paper and pencil activities and use some illustrations that are prone to induce or reinforce students' alternative conceptions. Therefore, the way textbooks address the GMO and TO theme needs to be improved.

Keywords: genetically modified organisms; science textbooks; secondary school; transgenic organisms

Context of the research

School science education should promote the development of informed, critical, active and responsible citizens. To succeed in doing so, it should address controversial socio-scientific issues (Reis, 2008; Oulton, Dillon, & Grace, 2004) such as the production and use of genetically modified organisms (GMO) and transgenic organisms (TO). Recent research on GMO and TO suggests that: the topic is not sufficiently addressed in schools (Matos, 2010) and has low emphasis in science syllabuses (Usak et al., 2009); students are poorly informed about the topic and hold alternative conceptions on these concepts (Matos, 2010; Carvalho, 2014; Ribeiro, Barone, & Behrens, 2016); textbooks that deal with these issues do not do it in the best way (Matos, 2010); these materials are not well accepted by the population, due to beliefs in possible risks for human health and the environment (Kim et al., 2018; Rzymski & Królczyk, 2016). However, it is worth noting that he greatest skepticism about GMO and TO production and use was expressed by farmers, doctors and teachers, while medical students and researchers showed enthusiasm (Rzymski & Królczyk, 2016) about them.

By addressing controversial issues like these, science education would increase students' motivation and engagement in learning, promote their understanding of the physical and natural world (Dolan, Nichols, & Zeidler, 2009), foster the development of their cognitive, social, moral, ethical and emotional skills (Tidemand & Nielsen, 2017) as well as their ability to take informed decisions (Tal & Kedmi, 2006). However, teachers can be expected to need some support to address these socially complex concepts properly in their classrooms.

Despite the ever increasing role of technology, the textbook remains one of the most frequently used science teaching resources in school science classes (Santos, 2001), by both students and teachers. Therefore, it can be expected that what students can learn about GMO and TO depends heavily on the way the assigned textbook deals with them. Despite the important role played by the textbook, research has shown that science textbooks are not always organized in such a way as to favour students' meaningful learning of science and that they may even induce or reinforce alternative conceptions in students' minds. This can be due to the reader's low reading competences, to the lack of objectivity of the text, to the quality of the illustrations as well as to the inadequate integration of both (Khine, 2013).

In a constantly changing society, teaching practices have to change too in order to keep fitting the ever new societal demands (Nieveen & Plomp, 2018). The same applies to syllabuses and textbooks, if they are to raise students' interest to learn and to offer support to teachers to promote their students' learning. As far as possible, a textbook should be consistent with the corresponding grade level syllabus demands (Tufekcic, 2012; Pingel, 2010), use language suitable to the target students, from a technical and a linguistic point of view, include activities for students to practice the knowledge they are expected to learn, adopt a friendly graphic design, as well as clear criteria for illustration inclusion. All these aspects - conceptual, methodological and graphical - are relevant and need to be well accomplished for a textbook to qualify as a good textbook.

Research aim

GMO and TO are quite new science topics which are socially relevant as they can have several implications for people and society. Textbooks should address them in a way as to facilitate low secondary school (7th to 9th grade) students' learning. However, research indicates that textbooks often address complex themes in a superficial way (Li et al., 2018), and do not devote the necessary space to controversial (Snyder & Broadway, 2004) and socio-scientific (Reis, 2004) topics. Thus, this research aims at finding out how six 9th grade Natural Science textbooks, used in Portuguese schools, approach the GMO and TO issues. The results may be useful for teacher education as they can show whether teachers can rely on the textbooks or whether they rather need to find strategies to overcome the risks for students' learning that are conveyed by an inadequate assigned textbook. Besides, it can raise textbook authors' awareness of textbook quality criteria and contribute to their improvement.

Literature review

GMO and TO concepts and students' alternative conceptions

According to article number 2 of the Portuguese Decree-Law 72/2003, a GMO corresponds to any organism other than human beings whose genetic material has been modified in a non-natural way, by crossing or natural recombination of genes. A TO is an organism in whose DNA one or more foreign genes (ie, from another organism) were inserted (Belzile, 2002; Corazza-Nunes et al., 2007). This means that all TO are GMO, but not all GMO are TO. An example of the latter is an organism that suffer genetic modification without receiving a gene from another organism.

Lack of information and uncertainty about the benefits and risks of GMO and TO have motivated numerous scientific, political and economic debates, which 21st century citizens should be able not only to follow but also to engage in in an informed way.

Opinions on GMO and TO production and use differ with regard to their health, environment, economic (Cui & Shoemaker, 2018), legal, political (Arcieri, 2015) and ethical implications (Ricroch, Guillaume-Hofnung, & Kuntz, 2018).

On one hand, it has been argued that the organisms that are at stake can be used for gene therapy, to produce reach nutritional components, vaccines and pest resistant agricultural products (Mackey, 2003; Zapata, 2007; Matos, 2010). On the other hand, risks for the environment have been pointed out. They include threat to biodiversity, due to competition between GM species and other species, increase of pest resistance to chemicals (Sehnal & Drobník (2009). Indirect negative effects of GMO have also been pointed out. They encompass raise of human resistance to antibiotics, increase of allergies due consumption of GM food and crushing of small traditional companies that cannot stand before the giant GM producers (Varzakas, Arvanitoyannis, & Baltas, 2007; Matos, 2010). Finally, some authors have put forward ethical arguments, that are based on the idea that man has no right to interfere, manipulate and modify nature at his will (Belzile, 2002; Matos, 2010; Zapata, 2007; Santos, 2006). For all these arguments and doubts, this is an excellent theme for students to actively engage in and to develop decision making competences and also to increase awareness of the provisional and dynamical character of scientific concepts and ideas.

In fact, there are not too many certainties about GMO and TO production and use. Arcieri (2015) even stated that long-term effects and consequences of GMO and TO production and use are yet difficult to predict. However, three main issues have been discussed (World Health Organization) without a consensus being reached. They are: their potential to provoke allergic reactions; the possibility that the genetic material transferred adversely affects human health; the possibility of migration of genes from GMO or TO into conventional crops or related wild species having indirect effect on food safety. Hence, some safety assessments on GMO and TO production and use were performed, on a case-by-case basis (WHO, 2014). These assessments focused on six aspects: toxicity; allergenicity; nutritional or toxic properties; stability of the inserted gene; nutritional effects associated with genetic modification; and any unintended effects, which could result from gene insertion. In fact, it should be noted that all GMO and TO, that are currently on the international market, have passed risk assessments conducted by national authorities (Arcieri, 2015).

In Portugal, discussions on GMO and TO are rare. However, the country has legislation based on the directive 2001/18/EC of the European Parliament and the Council. This directive settled strict rules on production and use of GMO and TO, concerning: principles for environmental risk assessment; post-market monitoring requirements, including the long-term effects associated with interaction with other GMO and TO and the environment; mandatory information to the public; obligation for Member States to implement labeling and traceability at all stages of market placing. In addition, it stated that: the first permissions to produce and use GMO and TO would be in force for a maximum of 10 years; consultation of the Scientific Committee(s) would be compulsory; consultation of the European Parliament in decisions authorizing the release of GMO and TO would be compulsory; the Council of Ministers would accept or reject, by qualified majority, a Commission proposal to authorize a GMO and/or TO. At present, a European Union moratorium is in force in Portugal, which prevents the release of new GMO and TO to be produced or sold.

The issues mentioned above, including those focusing on uncertainties, should be referred to in textbooks if they are to convey an accurate idea about the state of the art with regard to the available knowledge on GMO and TO. This would also be important foster the development of students' alternative conceptions towards scientifically accepted ones. Several alternative conceptions on GMO and TO have been identified. Some of them are as follows: GMO contain dangerous substances and their use can destroy human genes (Dawson, 2007; Usak et al., 2009); a GMO is the same as a TO (Corazza-Nunes et al., 2007; Santos, 2006; Matos, 2010); a GMO is the same as a clone (Matos, 2010); GMO are originated by DNA mutations (Carvalho, 2014); a TO is an organism obtained from a GMO (Matos, 2010); TO are GM food only (Matos, 2010); GMO are obtained by selective reproduction technique (Dawson, 2007), that is, by crossing two individuals with desirable characteristics.

As students possess a variety of alternative conceptions on GMO and TO it is important that, when addressing these topics at school, students' alternative conceptions are taken into account in order to develop their scientific literacy on these products. Textbooks could help if they address the theme not only accurately but also in a socially relevant way, and also if they, in some way, foster a discussion of the alternative conceptions that students may hold.

Activities in textbooks

A school textbook is an educational resource written to support the teaching and learning of a subject, and it includes teaching and learning activities, as well as assessment activities, which and reinterprets the school curriculum (Okeeffe, 2013). The Portuguese law (Decree-Law 47/2006, article 3 (b), of August 28) also acknowledges this definition of school textbook. Therefore, it is important that textbook authors follow the official curriculum in order to minimize the "expected gap between the prescribed and the implemented curriculum" (Leite, Morgado & Dourado, 2016, p. 526) and the textbook can be better designed to support students' learning. However, textbook authors and teachers may have different opinions on the curriculum and on how it encompasses some societal topics. Thus, knowing that the textbook remains the most widely used teaching resource by teachers and students (Santos, 2001; Snyder & Broadway, 2004), it can be argued that teaching socio-controversial issues requires the use of a textbook that deals with such issues adequately (Tufekcic, 2012). That is why it is important to analyze textbooks and to be aware that inadequate and inconsistent or scientifically incorrect information in textbooks can affect students' conceptions on science concepts (Irez, 2009; Devetak & Vogrinc, 2013).

A quality textbook must obey to some criteria, as follows: the content must be actual and scientifically correct; the content must be adequate to the target group of students; the textual component must be linguistically correct and appropriate; the textbook must contain motivational elements; the textbook must encourage active learning; the textbook must contain activities at different cognitive levels (Devetak & Vogrinc, 2013). So, a quality science textbook should include activities: that help students build new knowledge in articulation with previous knowledge, overcoming possible alternative conceptions; include references to phenomena of students' daily life; that include representations of phenomena to clarify abstract ideas; that foster the application of the students' knowledge in different contexts (Roseman, Stern, & Koppal, 2010).

Usually, science textbooks include a variety of activities, such as paper and pencil, laboratory, research, and simulation activities, like roleplaying, and others. Leite (1999) showed that the activities that appeared often in most Portuguese textbooks of the time were: paper and pencil activities, which include quantitative and qualitative questions; and that "students' ability to communicate science is hardly promoted, since textbooks seldom include activities with that purpose" (p. 85).

Bearing in mind the importance of communication in science, to both learn and disseminate knowledge, it can be argued that it is relevant to include communication targeted activities in school science textbooks. They would have a high level of openness to involve students actively in cooperative learning processes, and could assume the form of roleplaying (Bhattacharjee & Ghosh, 2013), problem-solving activities (Roberts, 2004), research oriented laboratory activities (Leite & Dourado, 2013) or field activities (Dourado & Leite, 2013), etc. In any case, the selection of the activities needs to be consistent with the content to the taught, the objectives to be attained, the resources available to fulfil them and the features of the students who are supposed to perform the activities.

Science textbook illustrations

The appropriateness of the illustrations is a key feature for a textbook to be considered a good textbook. Illustrations may have especially important added value when the content focuses on issues that are unfamiliar to the students for geographic or time reasons or even because of the too large or the too small scale of the facts or phenomena that are at stake (Khine, 2013).

As it was referred to above, a good textbook should show an adequate relationship between text and illustrations (Devetak & Vogrinc, 2013). Illustrations have the potential to stimulate students' curiosity and to promote science content learning (Cook, 2008; Tufekcic, 2012). Besides, they can also be used show the relationship between facts and concepts, to support the explanation of complex situations or processes and to summarize content (Yasar & Saramet, 2007). In addition, illustrations can perform decorative, representative, organizational and explanatory roles (Chen, 2017). Illustrations with decorative roles are very common but they often make it difficult to understand the content, and may even make the students feel confused or become deconcentrated (Jaeger & Wiley, 2014; Chen, 2017).

The impact of the illustrations on learning has been increasingly acknowledged a consequence of this being an increasing number of illustrations included in textbooks (Dimopoulos, Koulaidis, & Sklaveniti, 2003). The role of illustrations is becoming more and more relevant as science includes more and more abstract concepts, that can be better explained using illustrations which represent the concepts or their exemplars (Devetak & Vogrinc, 2013; Khine, 2013; Leite, Morgado, & Dourado, 2016). In fact, some authors (Peeck, 1993; Pozzer-Ardengi & Roth, 2004) sate that illustrations improve significantly textbooks quality as stimulating a variety of students sensory fields (Khine, 2013), and therefore facilitate the understanding of the contents presented by them.

Illustrations are a part of the textbook language used in science textbooks (Pozzer-Ardengi & Roth, 2004) that includes photographs, diagrams, tables, charts and drawings (Cook, 2008; Slough & McTigue, 2013). Research indicates that the most commonly used type of illustrations in secondary school textbooks is photographs (Pozzer & Roth, 2003). In some content areas, like biology, the high use of photographs may be due to the fact that they are the most realistic type of illustrations and therefore can be expected to be more useful for students.

Assuming that illustration have a role to play in students learning, it can be argued that the selection of the illustrations to be included in a textbook is an important task that should be guided by a few important criteria. The illustrations must: be of good graphic and scientific quality; contain potentially motivating elements for the students (including caricatures, comics, etc.); raise students' awareness of their prior knowledge; be well integrated into the text; be of different types, depending on the students and the features of the content to be addressed (Khine, 2013). In addition, the illustrations must be appropriate to the students' socio-cultural context (Bellocchi, King & Ritchie, 2016), so that they have, among others, the desirable motivating effect. Besides, as space in a textbook is limited and has costs, illustrations should focus on things that students are not familiar with instead of things that students already know from their environment or their daily life. The latter would not bring any additional educational value to the textbook.

Research methodology

The theme Genetically Modified Organisms (GMO) and Transgenic Organisms (TO) was chosen for analysis due to its controversial status and to the fact that it combines sub-micro and macro elements which may make illustrations relevant for learning. In basic education, this theme is addressed by 9th grade Natural Science textbooks. Thus, the 2017 editions of six 9th grade Natural Science textbooks (see Annex) which are currently used in Portuguese schools and accessible in the Braga municipal library, were selected for analysis. In all textbooks, the theme is related to the applications of genetic engineering and it is addressed in two to four pages, which shows that it is not given too much emphasis.

In order to collect the necessary data, content analysis on the parts of the textbooks dealing with GMO and TO was performed based on a checklist adapted from another one (Dourado, Morgado, & Leite, 2015; Leite, Morgado, & Dourado, 2016; Matos, 2010) available in the literature. Three dimensions of analysis were considered: conceptual, methodological and illustrtration dimensions. The first dimension focuses on the key concepts and includes the following sub-dimensions: GMO concept; TO concept; relationship between GMO and TO; GMO and TO production; GMO and TO in the legal framework; and advantages and disadvantages of the production and use of GMO and TO. The second dimension deals with the methodology used to present the theme and includes the following sub-dimensions: types of activities presented; demands on students' engagement; and potential of the activities to promote students' conceptual change. The third dimension of analysis focuses on the illustration used to present the theme and includes: types of illustrations; relationship between illustrations and text; and relationship between illustrations and students' alternative conceptions on the key concepts encompassed by the topic under question. Afterwards, issues to be analyzed within the scope of each sub dimensions were defined partly based on the literature and partly based on the material to be analyzed, so that the final set of items (or sub-sub dimensions) to be analyzed could fit the textbooks content. The sub dimensions and their sub-sub dimensions will be introduced in the next section of the paper.

As far as data to be collected is concerned, two categories were used for the items within the scope of the conceptual and the methodological dimensions: present (√) or absent (---). With regard to the items within the scope of the graphical dimension, after classification according to the type of graphical representation, absolute frequency was computed per item.

To increase reliability of the data, content analysis was performed by one of the authors and those instances perceived as difficult to classify were discussed with the other author, being the final decision taken by consensus. In addition, descriptions of some instances analyzed will be provided so that the reader can judge on the reliability of the data.

Findings

Conceptual dimension

As far as the *GMO concept* sub-dimension is concerned, Table 1 shows that all the six textbooks mention the acronym GMO and its meaning but only five of them mention the term genetically modified organism. Only three textbooks give a correct definition of GMO, which, consistent with the Portuguese law (Decree-Law 72/2003, article 2), to which a GMO is any organism (excluding human beings) whose genetic material has been modified in a non-natural way, by crossing and/or natural recombination of genes.

Regarding the sub-dimension *TO concept and its relation to GMO*, Table 1 shows that only three textbooks present the concept of TO and only two of them distinguish the concept of GMO and TO correctly given the school level that is at stake. Two of these textbooks (ST3 and ST5) sate that 'Genetically Modified Organisms (GMO) correspond to living beings whose genetic material has been manipulated in such a way as to favour a desired feature. If one or more genes from another species are introduced into the GMO, it is a transgenic', which is in accordance with the definition given above. The third textbook (ST2) gives incorrect definitions of the concepts and presents them as synonyms: "Genetically modified or transgenic seeds, to better resist pests, are now widely used in maize cultivation in Portugal." (p. 129) and "(…) for example, the production of transgenic foods or genetically modified foods can lead to biodiversity loss and to the creation of multi-resistant pests." (p. 130).

As shown by Table 1, there are *examples of GMO* mentioned in all but one (ST4) Textbooks. The most common examples, mentioned in three of the six textbooks, are: transgenic maize, transgenic soybean, golden rice, and bacterium for human insulin production. Except the last one, they fall into the area of the food industry.

Regarding the *Production of GMO and TO* sub-dimension, in three of the textbooks, it is explained based on technique of DNA recombination. However, only illustrations are provided; no verbal explanation of how it works is given. Besides, no information on large scale or industrial production is given too.

As far as the sub-dimension *Advantages and risks of GMO and TO production and use* is concerned, Table 1 shows that all but one (ST3) of the textbooks discuss advantages and disadvantages of the production and use of GMO and TO. Health benefits related to the production of more nutritious food (ST4 e ST6) and vaccines, drugs and hormones (ST4, ST5 and ST6) are mentioned. Some economic benefits associated to the increase of the resistance of organisms to pests, enabling longer periods without rotting (ST2 and ST4) as well as faster growing animals such as salmon (ST2), are presented too. ST1 addresses the advantages of the production and use of GMO and TO through an activity for the student to perform. Ecological disadvantages are explicitly mentioned only in ST2, ins association with possible loss of biodiversity and development of multi-resistant pests. Finally, it should be mentioned that all textbooks state briefly that there is no consensus on the production and use of GMO and TO and that there may be associated risks. However, risks to be taken into account are not mentioned. Therefore, textbooks tend to focus more on the advantages of GMO and TO production and use than on their disadvantages or possible risks and to underemphasise the doubts that still remain on them.

Finally, *Legal framework* sub-dimension regarding the production and use of GMO and TO, when referred, is absent (five textbooks) or insufficiently addressed (ST3) in the textbooks analysed. ST3 addresses it in the Portuguese context only (not considering the European one, for example) with a focus on the protection of individual's genetic information (Decree Law 12/2005, article 6 of January 26) without any explicit reference to the production and use of GMO and TO. ST3 provides a short document on the individual's genetic information and its use for hiring people for a particular job, as well as two excerpts from the Decree Law referred to above that spells out citizens' rights regarding the use of their genetic information. Even though these transcriptions focus on relevant issues, they are side issues of the main theme of the chapter.

Table1. The conceptual dimension of GMO and TO in Science Textbooks

Sub-Dimensions	**Items**	**Natural Sciences textbooks**					
		ST1	**ST2**	**ST3**	**ST4**	**ST5**	**ST6**
GMO concept	Refers the concept of GMO	√	---	√	√	√	√
	Refers the acronym of GMO	√	√	√	√	√	√
	Refers the meaning of the GMO acronym	√	√	√	√	√	√
	Gives a correct definition of GMO	√	---	√	---	√	---
TO concept and its relation with GMO	Presents the concept of TO	√	---	√	---	√	---
	Present the concept of GMO as a synonym of TO	---	√	---	---	---	---
	Distinguishes correctly GMO from TO	---	---	√	---	√	---
Examples of GMO mentioned in the Textbooks	Mentions cereals (corn, soy, rice)	√	√	---	---	√	√
	Mentions vegetables (potato, pea, tomato, carrot)	---	---	√	---	√	√
	Mentions fish (salmon)	---	√	---	---	---	---
	Mentions bacteria (*escherichia coli*, production of insulin)	√	√	√	---	√	√
	Mentions virus (Pexa-Vec)	√	---	---	---	---	---
Production of GMO and TO	Explains based on the recombination of DNA	---	---	√	√	√	---
Advantages and risks of GMO and TO production and use	Mentions Environmental advantages	√	---	---	---	---	√
	Mentions Health advantages	√	√	---	√	√	√
	Mentions Economic advantages	---	√	---	---	√	---
	Mentions Environmental risks	---	---	---	---	---	---
	Mentions Ecological risks	---	√	---	---	---	---
	Mentions Health risks	---	---	---	---	---	---
Legal framework	Refers the legal European framework related to GMO and TO	---	---	---	---	---	---
	Refers the legal Portuguese framework related to GMO and TO	---	---	√	---	---	---

Textbooks are expected to help students to overcame their alternative conceptions rather than to induce scientifically non-accepted ideas on them. However, ST2, approaches GMO and TO by relating them to transgenic food and genetically modified seeds. Transgenic food and genetically modified seeds are not synonymous of GMO and TO and using them as synonymous, as ST2 does ("Genetically modified or transgenic seeds […]", p. 129), can induce alternative conceptions in students. Besides, students can associate GMO and TO with the food industry only, discarding other industries that also deal with GMO and TO.

About GMO, ST4 states that: "Genetically modified organisms (GMO) include plants that have been improved to produce more nutritious food, to resist pests or to withstand long periods of storage without rotting" (p.233). This definition may induce students' alternative conceptions, as it may lead them to associate GMO and TO with plants only.

Finally, ST1 relates the concepts of GMO and TO is an unclear way, bay stating that GMO is a "Designation given to an organism whose genetic material has been manipulated. If the manipulated genetic material comes from another species, the GMO is called transgenic" (p. 230). This statement may induce alternative conceptions in the students because it seems that a TO is obtained when the genetic material inserted in a certain organism comes from another species that had been previously genetically manipulated.

Therefore, for what they say and for the way they say it, three out of the six textbooks analysed can induce alternative conceptions in students instead of helping them to overcome the alternative conceptions that they may have constructed outside school.

Methodological dimension

As far as the *Types of activities* used to approach GMO and TO Table 2 shows that two (ST4 and ST5) out of the six textbooks do not propose any activities. The other four textbooks (ST1, ST2, ST3 and ST6) include paper and pencil activities. Only one of them (ST6) suggests a roleplaying activity and another one (ST3) proposes a teamwork activity to be followed by a discussion.

The paper and pencil activities appears after the concept presentation, as 'application' exercises. Therefore, they do not encourage too much analytical thinking. In fact, one of the paper and pencil activities included in ST1 presents three short texts titled "*E. coli* genetically engineered to decontaminate mercury water", "Golden rice can help combat vitamin A deficiency" and "*Pexa-Vec* destroys cancer cells", and asks students to answer the following questions: "Refer three examples of applications of genetics in society"; "Mention the advantage of using transgenic E. coli in the decontamination of environments rich in mercury"; "Indicate the main nutritional characteristic of 'golden rice'"; and "Refer how the Pexa-Vec virus acts on cancer cells" (p.230). To answer these questions, students just need to read the texts and find the information there.

The teamwork activity followed by discussion that is proposed by ST3 is presented as a task, integrated into a set of questions, as follows: "Conduct a group research on the benefits and limitations/risks of genetic testing. Afterwards, discuss the information gathered in a large group (class)."(p. 231).

The roleplaying activity involves a "Decision Making" process on bioethical problems arising from the application of genetics in society - the golden rice crop. It starts with three news excerpts, for and against the cultivation of golden rice and students are given specific roles for the purpose of argumentation and voting: a scientist, a doctor, a father and a mother, a businessman and a representative of the populations. After the role playing, students are asked to write a news on the roleplaying and to publish it in the school newspaper. Although roleplaying activities have the potential to promote students' ability to reason and motivates critical thinking and reflection (Bhattacharjee & Ghosh, 2013), the activity proposed in the textbook should have more guidance for students and teachers, namely with regard to information to be used to build arguments from.

Table 2. The methodological dimension of GMO and TO in Science Textbooks

Sub-Dimensions	**Items**	**Natural Sciences textbooks**					
		ST1	**ST2**	**ST3**	**ST4**	**ST5**	**ST6**
Types of activities presented	Paper and pencil activities	√	√	√	---	---	√
	Roleplaying	---	---	---	---	---	√
	Teamwork with discussion	---	---	√	---	---	---
Demands of the activities in terms of students' engagement	High engagement of students	---	---	---	---	---	√
	Medium engagement of students	---	---	√	---	---	---
	Low engagement of students	√	√	---	√	√	---
Potential of the activities for students' conceptual change promotion	Able to promote conceptual change	---	---	√	---	---	√

Given the types and features of the activities included in the textbooks, the *Demands of the activities in terms of students' engagement*, are low, except in the case of teamwork and roleplaying. Textbook that suggests a teamwork activity (ST3) was rated as favouring student's medium engagement in learning content as if students are to succeed they have to combine conceptual, methodological, as well as relational and communication skills. ST6, that suggests a roleplaying activity, was rated as promoting student's high engagement because it requires students to perform the activities mentioned for the team work activity and also to act as another person, with the features of the role he/she is invites to personify.

Finally, with regard to the last sub-dimension, *Potential of the activities to promote students' conceptual change*, no paper and pencil activity was designed to elicit a students' alternative conception or was focusing on the discussion of an alternative conception. Textbooks that suggest roleplaying and teamwork activities may favour conceptual change in students if the teachers can monitor the ideas that students verbalize and can make them perceive their possible weaknesses or inconsistencies.

Illustrations dimension

Table 3 shows that, in some of the textbooks, the number of illustrations is small, as it could be expected due to the reduced number of pages devoted to the contents that are at stake. The total number of illustrations is as follows: ST1 - 3; ST2 - 5; ST3 and ST4 - 6 illustrations each; ST5 - 19 illustrations; and ST6 - 13 illustrations. It should be noted that ST5 has the highest number of illustrations because it uses the same illustration seven times to beautify one page.

Table 3. The illustrations dimension of GMO and TO in Science Textbooks (f)

Sub-Dimensions	**Items**		**Natural Sciences textbooks**					
			ST1	**ST2**	**ST3**	**ST4**	**ST5**	**ST6**
Types of illustrations	Real photographs		2	4	0	2	8	5
	Drawings-like photographs		0	0	3	1	6	3
	Photographs combined with other elements		0	1	0	1	0	1
	Drawings-like photographs combined with other elements		1	0	2	2	5	4
	Flow chart		0	0	1	0	0	0
Relationship between illustrations and text	Illustrations are related to the content	Explicitly mentioned	0	0	1	0	10	8
		Not explicitly mentioned	1	4	3	4	0	1
	Illustrations are not related to the content	Simply add new information	1	0	1	1	2	2
		Work as a background to beautify the page	1	1	1	1	7	2
Relationship of illustrations with students' alternative conceptions	Susceptible of promoting the formation of alternative conceptions on GMO and TO		1	2	2	2	5	6
	Susceptible of leading students to overcome alternative conceptions on GMO and TO		0	2	0	0	0	0
	Neutral		2	1	4	4	14	7

As far as the *Types of illustrations* sub-dimension is concerned, photographs and drawings-like photographs are the most frequent types of illustrations. These results may suggest that textbook authors are trying to bring reality into learning process.

Regarding the *Relationship between illustrations and text* sub-dimension, some illustrations are related to the content and others are not. Only three textbooks (ST3, ST5 and ST6) explicitly mention a few illustrations in the text, even though these are not fully explored but only mentioned into brackets. The remaining three textbooks (ST1, ST2 and ST4) make no explicit mention to illustration in the text even though they present a few illustrations that the reader can easily perceive that are related to issues being dealt with.

Illustrations that do not relate to the content may simply add more information to what is referred in text or to have not apparent relation with the content being presented. 'Drawings-like photographs combined with other elements' usually add new information to what is said in the text.

All textbooks included illustrations for the purpose of beautification of the book page, ie, illustrations that serve to make the page visually appealing, without conveying any informative message. These illustrations may be unnecessary from an educational point of view (Perales, 2008; Leite, Morgado & Dourado, 2016) as they may even reduce the readability of the text by introducing 'visual noise'.

As far as the sub-dimension *Relationship of illustrations and students' alternative conceptions on issues that are at stake*, Table 3 shows that all textbooks include illustrations that can lead to the formation of alternative conceptions by the students that use them. For example: ST2, ST3, ST4 and ST5 include illustrations referring to the cloning process (Dolly sheep cloning and therapeutic cloning) with no clear graphic separation between the theme of GMO and TO and the theme of cloning. This fact may induce alternative conceptions in students as they may assume that GMO and TO are the similar to clones.

ST4 uses terms such as "transgenic foal" in the captions of some illustrations. As the meaning of this term is not explained in the text, it may raise doubts to students or even lead them to confound it with the scientifically accepted concept of TO. The same textbook presents a scheme of the DNA recombination technique without verbal support, which can lead students to confound simple DNA recombination with the concepts under question.

ST1 provides an illustration of the process of medically assisted reproduction without clearly separating it from the GMO and TO topic. This may lead to the formation of alternative conceptions, as students may consider that medically assisted reproduction is a process that involves GMO and TO.

In four out of six textbooks (ST3, ST4, ST5 and ST6), the size ratio between the organism (enzymes, genes, cell, etc.) real size and its illustrations size may lead to the formation of alternative conceptions in the students, as neither illustrations are to scale nor there is any note on this issue.

Finally, ST6 includes an illustration on the use of E. coli bacteria for the production of drugs against Hepatitis C and haemophilic, but it does not clearly explain how the genetic modification made to obtain this drug is done. The way the textbook does it may lead to the idea that the bacterium is inserted into the medicine itself, which is not true.

Conclusions and implications

This empirical study aimed at finding out how six 9th grade Natural Science textbooks, used in Portuguese schools, approach the GMO and TO issues, focusing on three overarching issues: a conceptual, a methodological and a graphical dimension.

Regarding conceptual issues, results show that textbooks: present the concept of GMO and TO, but do not always relate them to each other; the process of GMO and TO production is hardly addressed and when it is addressed this is done by illustrations only; the examples of GMO and TO provided are not enough to demonstrate the complexity of the subject; the advantages of producing and using GMO and TO are given more emphasis than their risks. Results relative to methodological issues show that textbooks tend to describe content and to provide few activities for students, being most of them low cognitive demand paper and pencil activities. Only one textbook presents a teamwork activity followed by a discussion and another textbook presents a roleplaying activity. With regard to the illustrations issue, results show that; photographs and drawings-like photographs are the most frequently used type of illustration; illustrations are rarely explicitly mentioned in the text; and some of them are prone to induce or reinforce students' alternative conceptions. In summary, it can be stated that the Natural Science textbooks analysed do not adequately address the GMO and TO socio-scientific controversial theme.

The results obtained compare to those reached by other authors that analysed textbooks from a conceptual (Corazza-Nunes et al., 2007; Dawson, 2007; Matos, 2010; Santos, 2006;) or a methodological (Bhattacharjee & Ghosh, 2013; Leite, 1999) point of view. They also compare to those obtained by authors that analysed the illustrations (Cook, 2008; Dourado, Morgado, & Leite, 2015; Leite, Morgado, & Dourado, 2016; Pozzer & Roth, 2003; Slough & McTigue, 2013) included in textbooks.

Thus, it can be argued that Natural Science textbooks need to be carefully reviewed in order to address this theme in a more effective way. Besides, if teachers want to approach the topic properly and give it the attention it may deserve, they need to go far beyond what textbooks suggest. They need to find better approaches to make their students scientifically informed and aware of their responsibilities as educated citizens. Given the newness of the theme, approaching it properly may require teachers to find ways to develop their own knowledge and skills on these issues.

This study focused on Portuguese basic education textbooks only. Other studies should be carried out in Portugal and in other countries in order to better understand how and why textbooks authors, teachers and students deal with this theme the way they do and how can it, and other socio-scientific themes, be improved in science textbooks for the benefit of citizenship education.

Acknowledgements

This research was funded by CIEd - Research Centre on Education, project UID/CED/01661/2019, Institute of Education, University of Minho, through national funds of FCT/MCTES-PT

References

Arcieri, M. (2015). Spread and potential risks of genetically modified organisms. *Agriculture and Agricultural Science Procedia, 8,* 552-559.

Bellocchi, A., King, D., & Ritchie, S. (2016). Context-based assessment: Creating opportunities for resonance between classroom fields and societal fields. *International Journal of Science Education, 38*(8), 1304-1342.

Belzile, F. (2002). Transgenic, transplastomic and other genetically modified plants: A Canadian perspective. *Biochimie, 84,* 1111-1118.

Bhattacharjee, S., & Ghosh, S. (2013, April). *Usefulness of role-playing teaching in construction education: A systematic review*. Paper presented at 49th ASC Annual International Conference Proceedings. California.

Carvalho, L. (2014). *O ensino da biotecnologia com recurso a um modelo de Ensino Orientado* *para a Mudança Concetual: uma abordagem centrada no tema dos Organismos Geneticamente Modificados e Transgénicos*. (Masters Teaching Practice Report). University of Minho, Braga.

Chen, X. (2017). A comparative study of visual representations in conventional, digitized and interactive high school science textbooks. *Journal of Visual Literacy*, *36*(2), 104-122.

Cook, M. (2008). Students' comprehension of science concepts depicted in textbook illustrations. *Electronic Journal of Science Education, 12*(1), 1-14.

Corazza-Nunes, M., Pedrancini, V., Fujii, R., Nunes, W., Moreira, A., & Galuch, M. (2007, November). O que conhecem os estudantes do ensino médio e ensino superior sobre transgênicos. *VI Encontro Nacional de Pesquisa em Educação em Ciências*. Florianópolis, Associação Brasileira de Pesquisa em Educação em Ciências.

Cui, K., & Shoemaker, S. (2018). Public perception of genetically-modified (GM) food: A nationwide Chinese consumer study. *Science of Food 2*(10), 1-8.

Dawson, V. (2007). An exploration of high school (12-17 year old) students' understandings of, and attitudes towards biotechnology processes. *Research in Science Education*, *37*(1), 59-73.

Devetak, I., & Vogrinc, J. (2013). The criteria for evaluating the quality of the science textbooks. In M. Khine (Ed.), *Critical analysis of science textbooks: Evaluating instructional effectiveness* (pp. 3-15). London: Springer.

Dimopoulos, K., Koulaidis, V., & Sklaveniti, S. (2003). Towards an analysis of visual images in school science textbooks and press articles about science and technology. *Research in Science Education 33*, 189-216. The Netherlands: Kluwer Academic Publishers.

Dolan, T., Nichols, B., & Zeidler, D. (2009). Using socio-scientific issues in primary classrooms. *Journal of Elementary Science Education, 21*, 1-12.

Dourado, L. & Leite, L. (2013). Field activities, science education and problem-solving. *Procedia - Social and Behavioral Sciences, 106*, 1232-1241.

Dourado, L., Morgado, S., & Leite, L. (2015). Photographs as mediating tools between science knowledge and the real world. *The Turkish Online Journal of Educational Technology*, *Special Issue 2 for INTE 2015*, 426-435.

Irez, S. (2009). Nature of science as depicted in Turkish biology textbooks. *Science Education, 93*(3), 422-447.

Khine, M. (2013). *Critical analysis of science textbooks: Evaluating instructional effectiveness*. London: Springer.

Kim, N., Hwang, J., Lee, H., Song, M., Kang, Y., & Rhee, M-S (2018). Strategic approaches to communicating with food consumers about genetically modified food. *Food Control, 92,* 523-531.

Jaeger, A., & Wiley, J. (2014). Do illustrations help or harm metacomprehension accuracy?. *Learning and Instruction, 34*, 58-73.

Leite, L. (1999). Heat and temperature: An analysis of how these concepts are dealt with in textbooks. *European Journal of Teacher Education, Abingdon, 22*(1), p. 75-88.

Leite, L., & Dourado, L. (2013). Laboratory activities, science education and problem-solving skills. *Procedia - Social and Behavioral Sciences 106*, 1677-1686.

Leite, L., Morgado, S., & Dourado, L. (2016). Contextualized science teaching: The contribution of photographs included in school science textbooks. *Turkish Online Journal of*

Educational Technology, Special Issue for INTE 2016, 524-537.

Li, X., Wang, L., Shen, J., Wang, J., Hu, W., Chen, Y., & Tian, R. (2018). Analysis and comparison of scientific inquiry activities in eight-grade physics textbooks in China. *Journal of Baltic Science Education, 17*(2), 229-238.

Mackey, M. (2003). The developing world benefits from plant biotechnology. *Journal of Nutrition Education and Behavior*, 35(4), 210-214.

Matos, L. (2010). *A problemática dos organismos geneticamente modificados e a formação científica do cidadão comum*. (Masters Dissertation). University of Minho, Braga.

Nieveen, N., & Plomp, T. (2018). Curricular and implementation challenges in introducing twenty-first century skills in education. In E. Care, P. Griffin, & M. Wilson (Eds.), *Assessment and teaching of 21st century skills* (pp. 259-276). Switzerland: Springer International Publishing

Okeeffe, L. (2013). A framework for textbook analysis. *International Review of Contemporary Learning Research, 2*(1), 1-13.

Oulton, C., Dillon, J., & Grace, M. (2004). Reconceptualizing the teaching of controversial issues. *International Journal of Science Education, 26*(4), 411-423.

Peeck, J. (1993). Incrasing picture effects in learning from illustrated texts. *Learning and Instruction, 3*, 227-238.

Perales, F. (2008). La imagen en la enseñanza de las Ciencias: Alguns resultados de investigación en la Universidad de Granada, España. *Formación Universitaria, 1*(4), 13-22.

Pingel, F. (2010). *UNESCO guidebook on textbook research and textbook revision*. Paris: United Nations Educational, Scientific and Cultural Organization.

Pozzer, L., & Roth, W. (2003). Prevalence, function, and structure of photographs in high school biology textbooks. *Journal of Research in Science Teaching, 40*(1), 1089-1114.

Pozzer-Ardenghi, L., & Roth, W. (2004, April). *Students' interpretation of photographs in high school biology textbooks.* Paper presented at the annual meeting of the National Association for Research in Science Teaching, Vancouver, BC.

Reis, P. (2004). *Controvérsias sociocientíficas: Discutir ou não discutir? Percursos de aprendizagem na disciplina de Ciências da Terra e da Vida* (Doctoral Dissertation non published), Faculdade de Ciências da Universidade de Lisboa.

Reis, P. (2008). *A escola e as controvérsias sociocientíficas: Perspectivas de alunos e de professores*. Lisboa: Escolar Editora.

Ribeiro, T., Barone, B., & Behrens, J. (2016). Genetically modified foods and their social representation. *Food Research International 84*, 120-127.

Ricroch, A., Guillaume-Hofnung, M., & Kuntz, M. (2018). The ethical concerns about transgenic crops. *Biochemical Journal 475*, 803-811

Roberts, R. (2004). Using different types of practical within a problem-solving model of science. *School Science Review, 85*(312), 113-119.

Roseman, J., Stern, L., & Koppal, M. (2010). A method for analyzing the coherence of high school biology textbooks. *Journal of Research in Science Teaching, 47*(1), 47-70.

Rzymski, P. & Królczyk, A. (2016). Attitudes toward genetically modified organisms in Poland: to GMO or not to GMO?. *Food Sec. 8*, 689-697.

Santos, E. (2006). *Ensino de ciências e literacia científica: O caso dos organismos geneticamente modificados.* (Masters Dissertation non published). Universidade de Aveiro, Aveiro.

Santos, M. (2001). *A cidadania na "voz" dos manuais escolares: O que temos? O que queremos?*. Lisboa: Livros Horizonte.

Sehnal, F., & Drobník, J. (2009). *White book genetically modified crops: Scientific opinion of czech researchers working with GMO*. Praga: Biology Centre of the Academy of Sciences of the Czech Republic.

Slough, S., & McTigue, E. (2013). Development of the graphical analysis protocol (GAP) for eliciting the graphical demands of science textbooks. In M. Khine (Ed.), *Critical analysis of science textbooks* (pp.17-30). Dordrecht: Springer.

Snyder, V., & Broadway, F. (2004). Queering High School Biology Textbooks. *Journal of Research in Science Teaching, 41*(6), 617-636.

Tal, T., & Kedmi, Y. (2006). Teaching socioscientific issues: Classroom culture and students' performances. *Cultural Studies of Science Education, 1*, 615-644.

Tidemand, S., & Nielsen, J. (2017). The role of socioscientific issues in biology teaching: From the perspective of teachers. *International Journal of Science Education, 39*(1), 44-61.

Tufekcic, A. (2012). Basic determinants of producing a modern textbook. *Metodicki obzori 15, 7*(2), sp.

Usak, M., Erdogan, M., Prokop, P., & Ozel, M. (2009). Biotechnology education - high school and university students' knowledge and attitudes regarding biotechnology: a Turkish experience. *Biochemistry and Molecular Biology Education, 37*(2), 123-130.

Varzakas, T., Arvanitoyannis, I., & Baltas, H. (2007). The politics and science behind GMO acceptance. *Critical Reviews in Food Science and Nutrition, 47*(4), 335 - 361.

Yasar, O., & Seremet, M. (2007). A comparative analysis regarding pictures included in secondary school geography textbooks taught in Turkey. *International Research in Geographical & Environmental Education, 16*(2), 157-188.

Zapata, F. (Ed.) (2007). *Fundamentos y casos exitosos de la biotecnologia moderna*. Cidade do México: El Colegio Nacional.

Annex

Science Textbooks analysed

Antunes, C., Bispo, M., & Guindeira, P. (2017). *Descobrir a Terra*. Ciências Naturais (9.º ano). Porto: Areal Editores, S.A.

Campos, C., & Dias, M. (2017). *Terra CN*. Ciências Naturais (9.º ano). Lisboa: Texto Editores, Lda.

Costa, I., Barros, J., Motta, L., Viana, M., & Santos, R. (2017). *Viva a Terra!*. Ciências Naturais (9.º ano). Porto: Porto Editora.

Moreira, J., Sant´Ovaia, H., &Pinto, V. (2017). *Compreender o corpo humano 9*. Ciências Naturais (9.º ano). Porto: Areal Editores, S.A.

Pereira, C., Ramos, J., Machado, J., & Brazão, R. (2017). *Exploratório*. Ciências Naturais (9.º ano). Lisboa: Raiz Editora.

Salsa, J., & Cunha, R. (2017). *CienTIC*. Ciências Naturais (9.º ano). Porto: Porto Editora.

University of Minho